

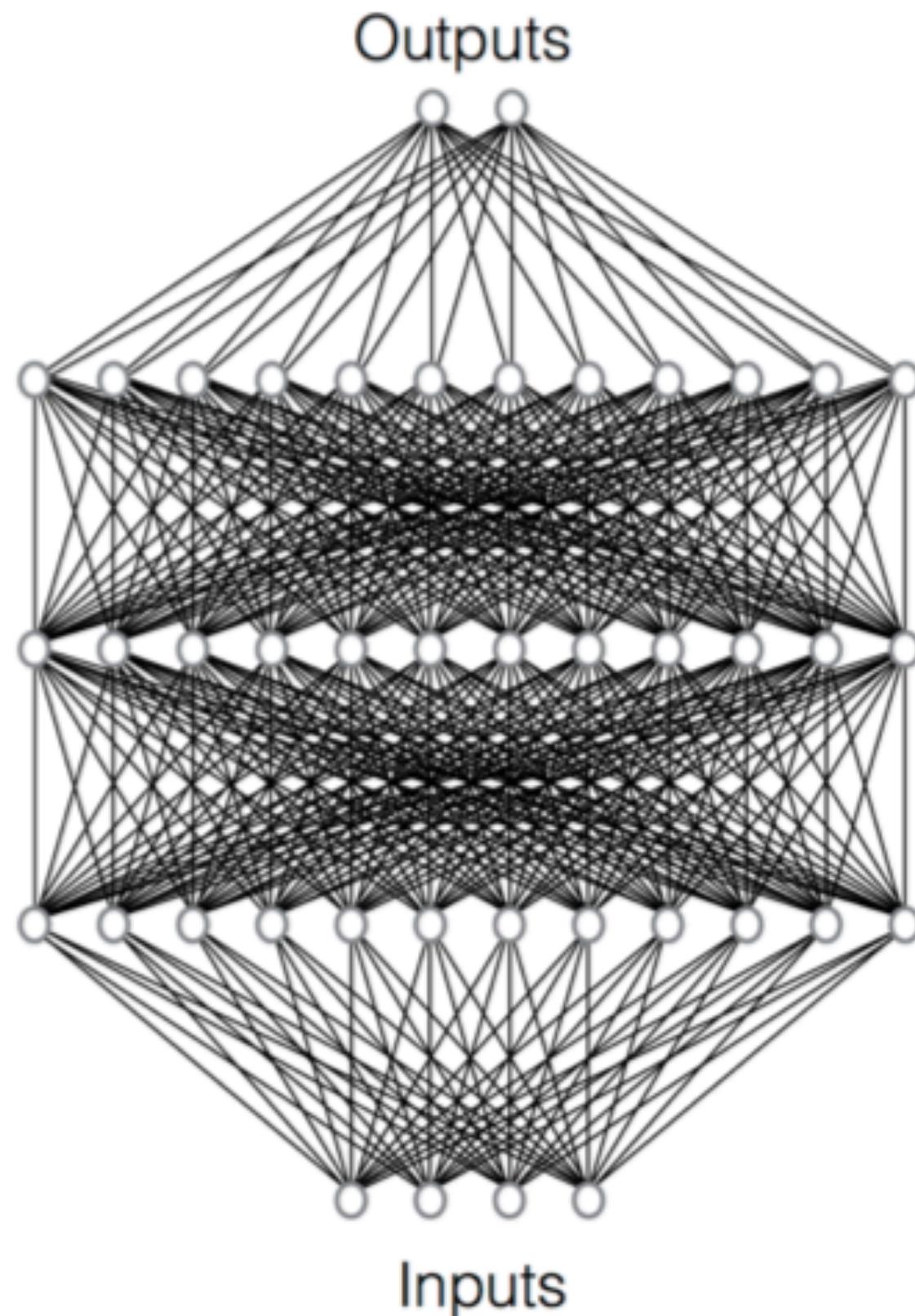


# Efficient Distributed Learning via Independent Subnet Training: Results and Trends

Anastasios Kyrillidis  
Rice CS

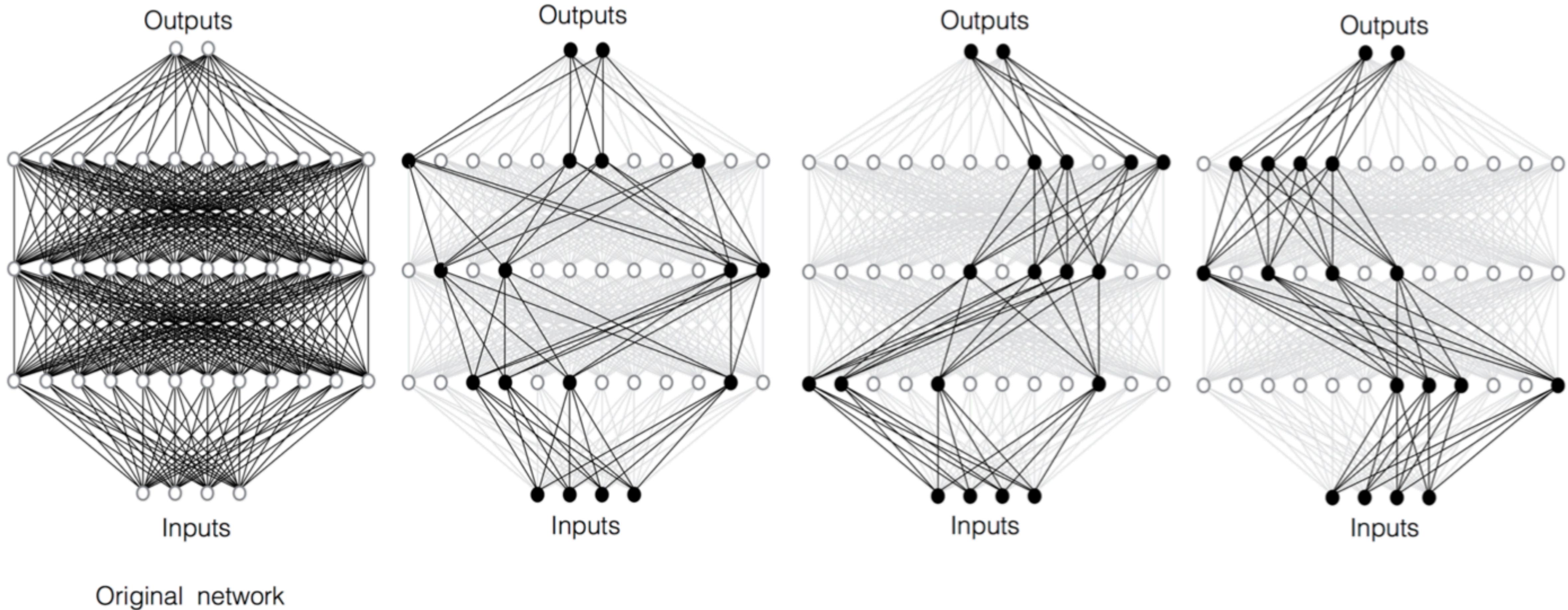
Joint work with: Binhang Yuan, Chen Dun, Cameron Wolfe, Fangshuo Liao,  
Qihan Wang, Yuxin Tang, Erdong Hu, Jingkang Yang,  
Santiago Segarra, Dimitris Dimitriadis, Chris Jermaine

# Independent Subnet Training: NN decomposition

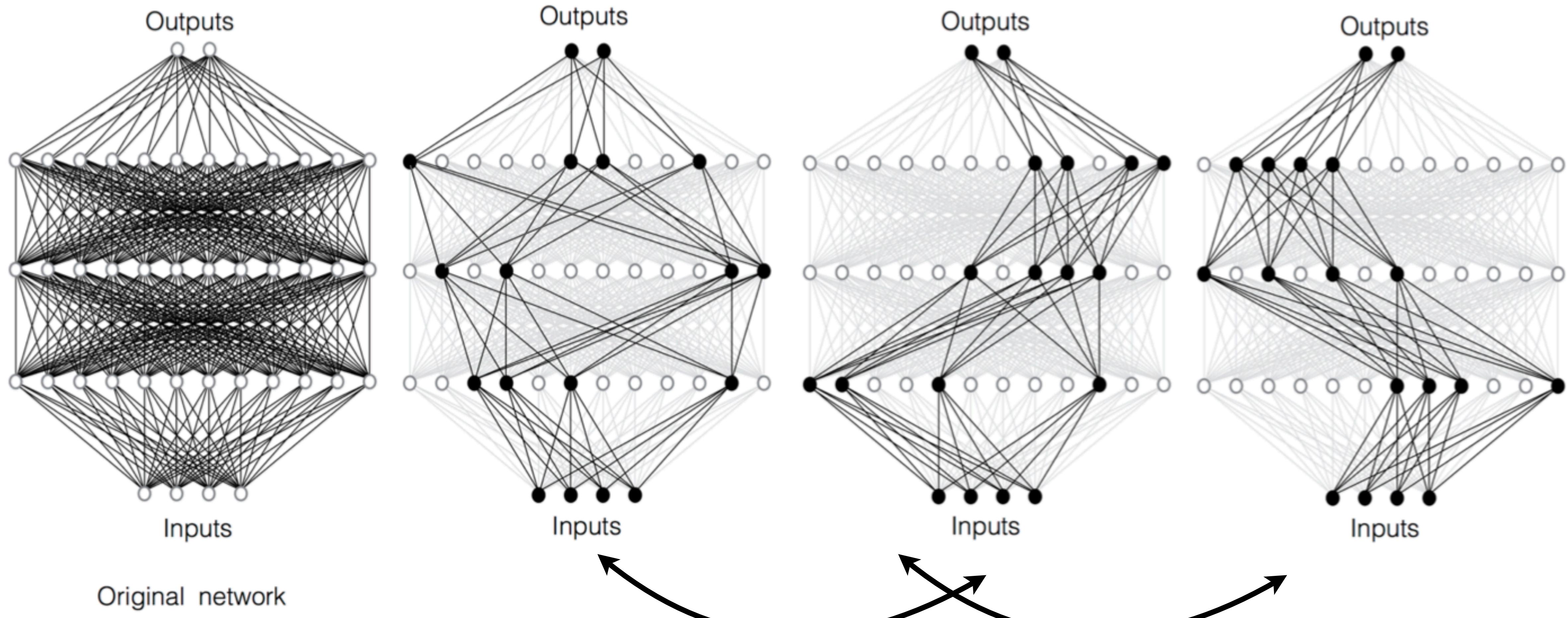


Original network

# Independent Subnet Training: NN decomposition



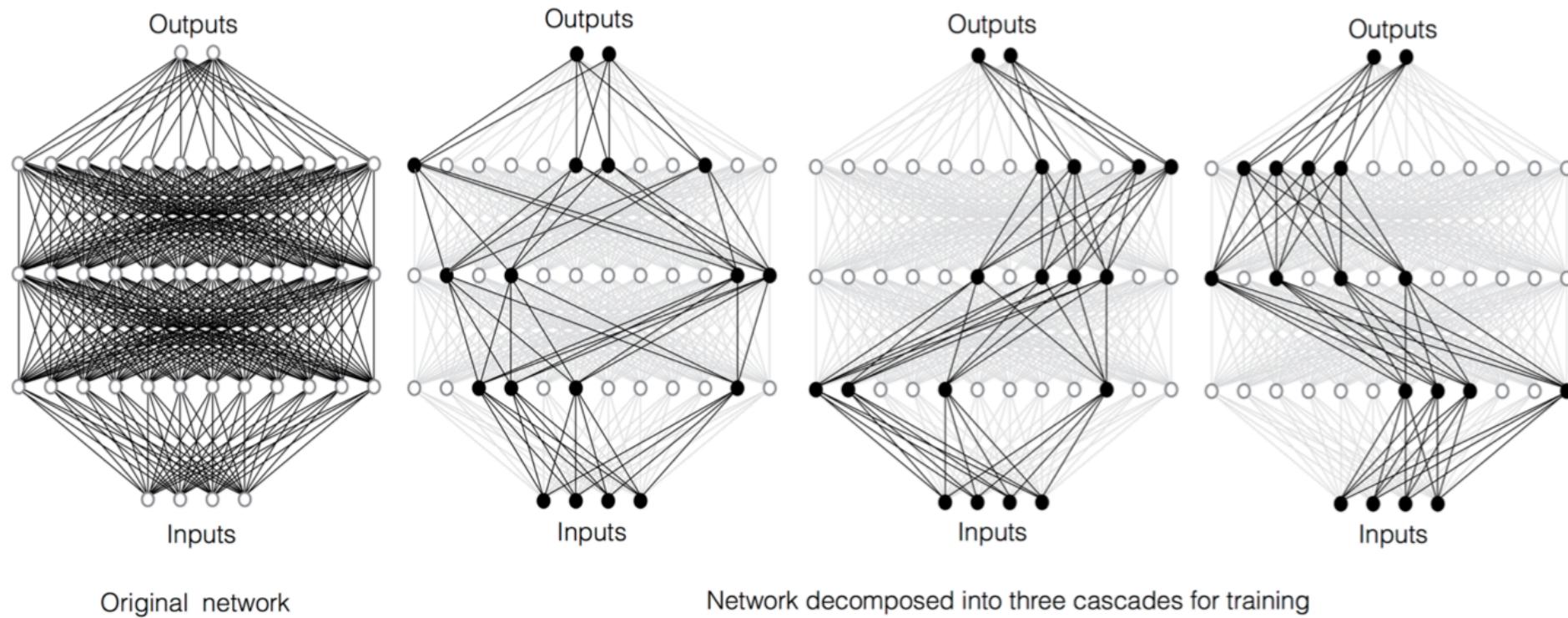
# Independent Subnet Training: NN decomposition



**Union of neurons make original network**  
(Note: union of parameters do not make original network necessarily)

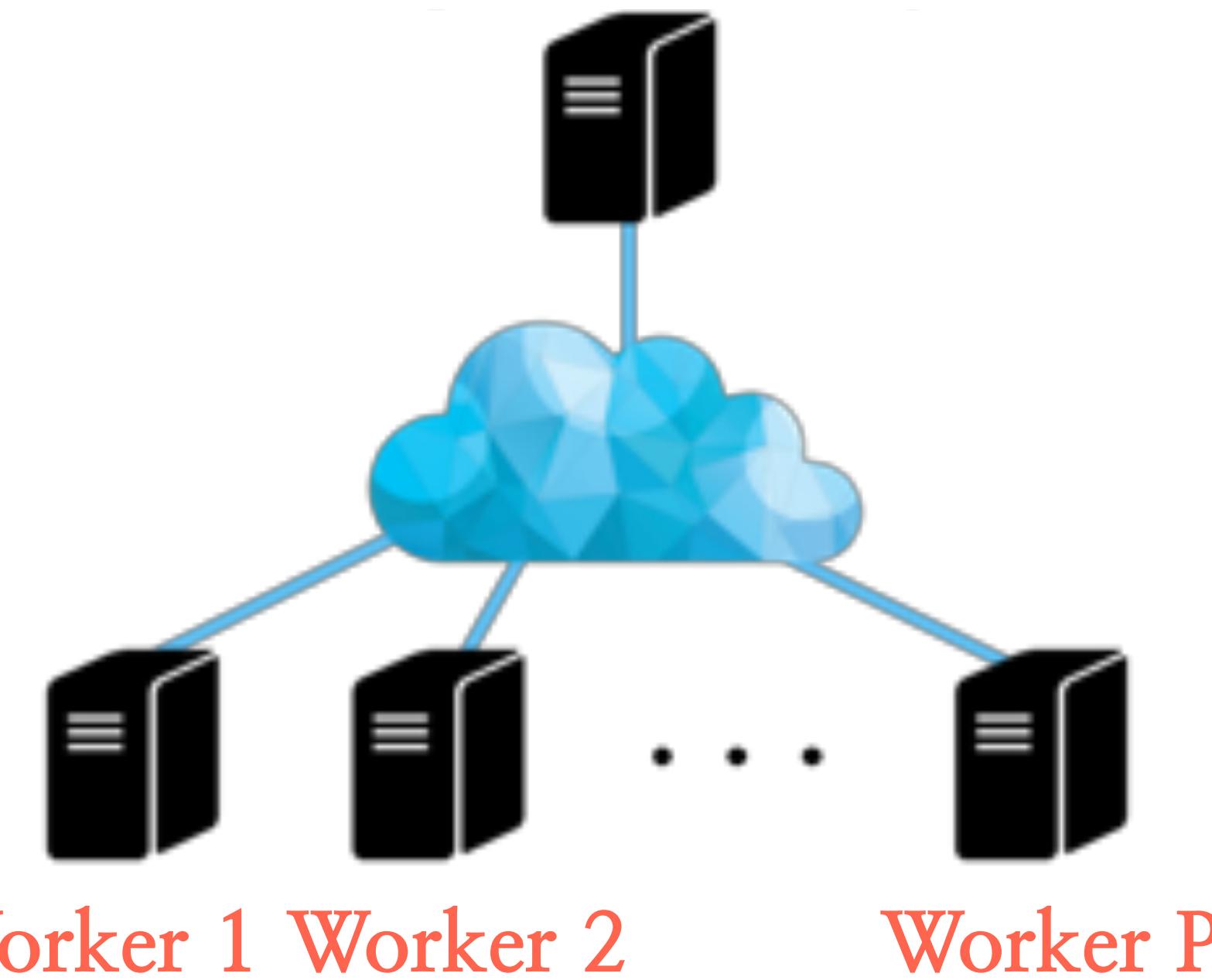
# Independent Subnet Training: NN distr. training

How to decompose a NN:

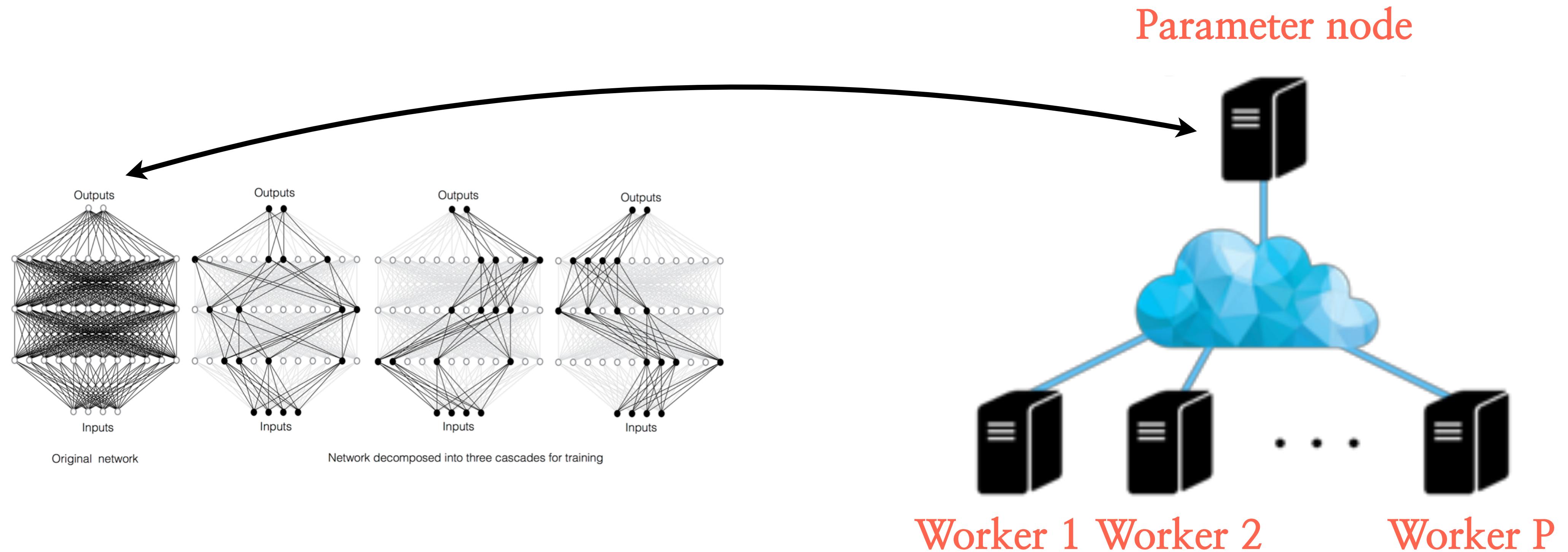


How to train NN in a distributed fashion:

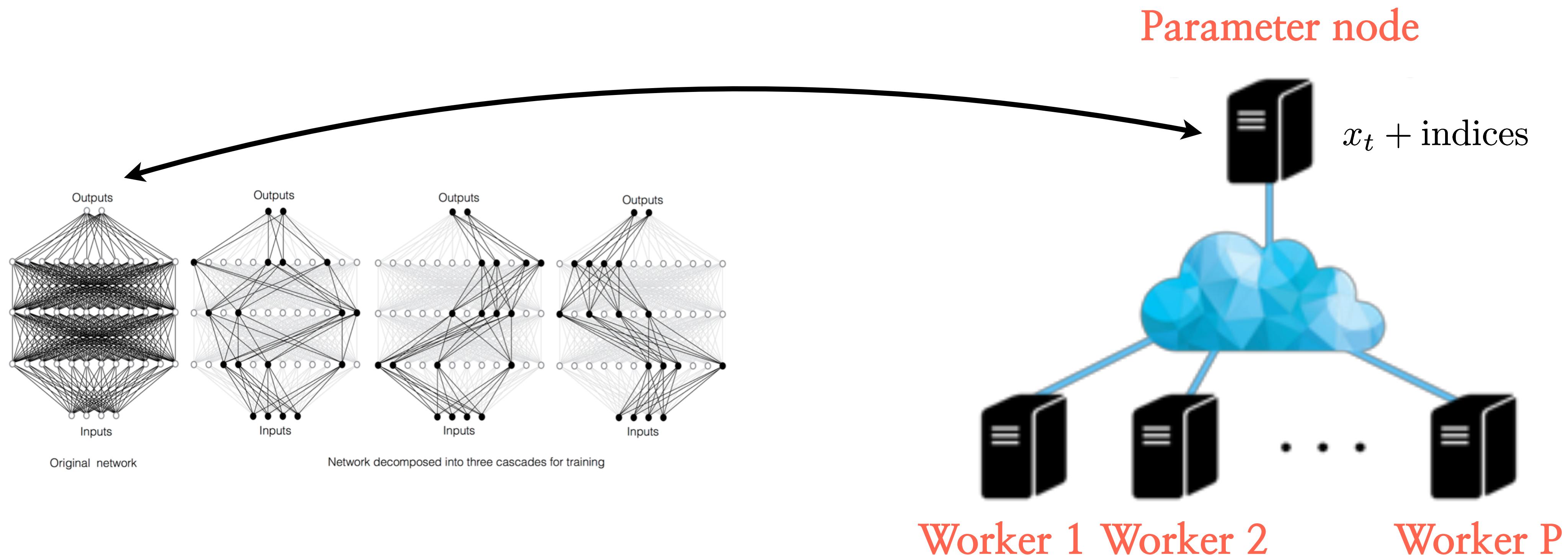
Parameter node



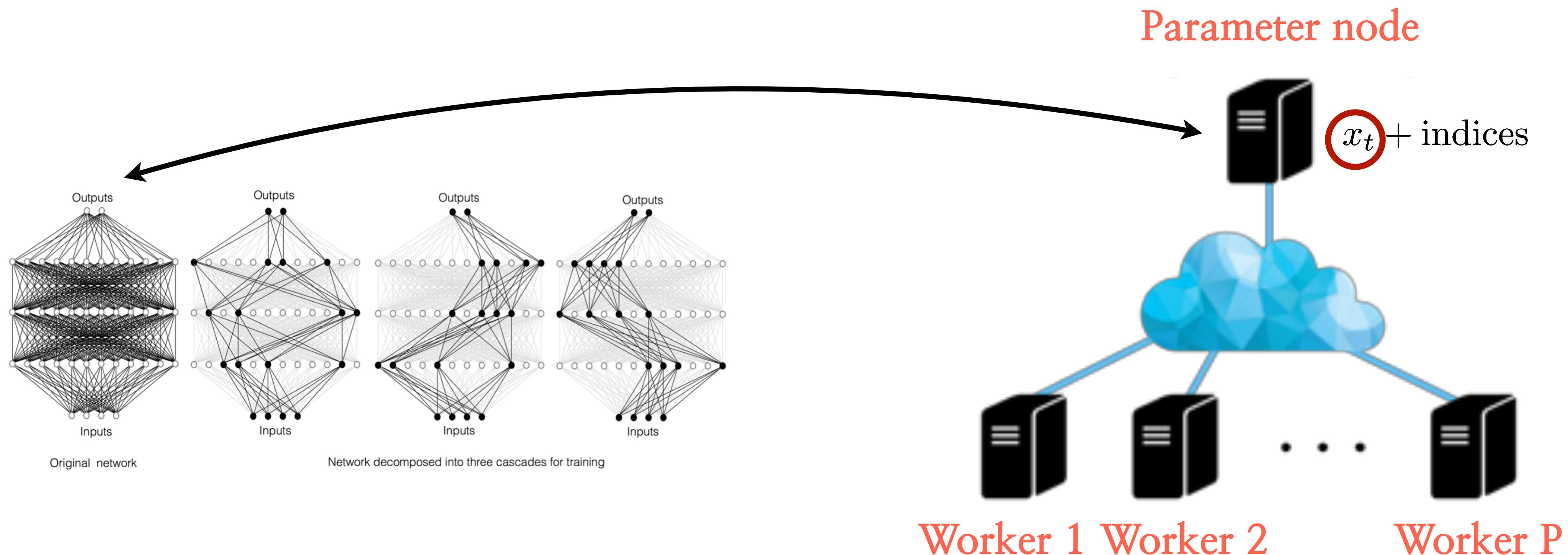
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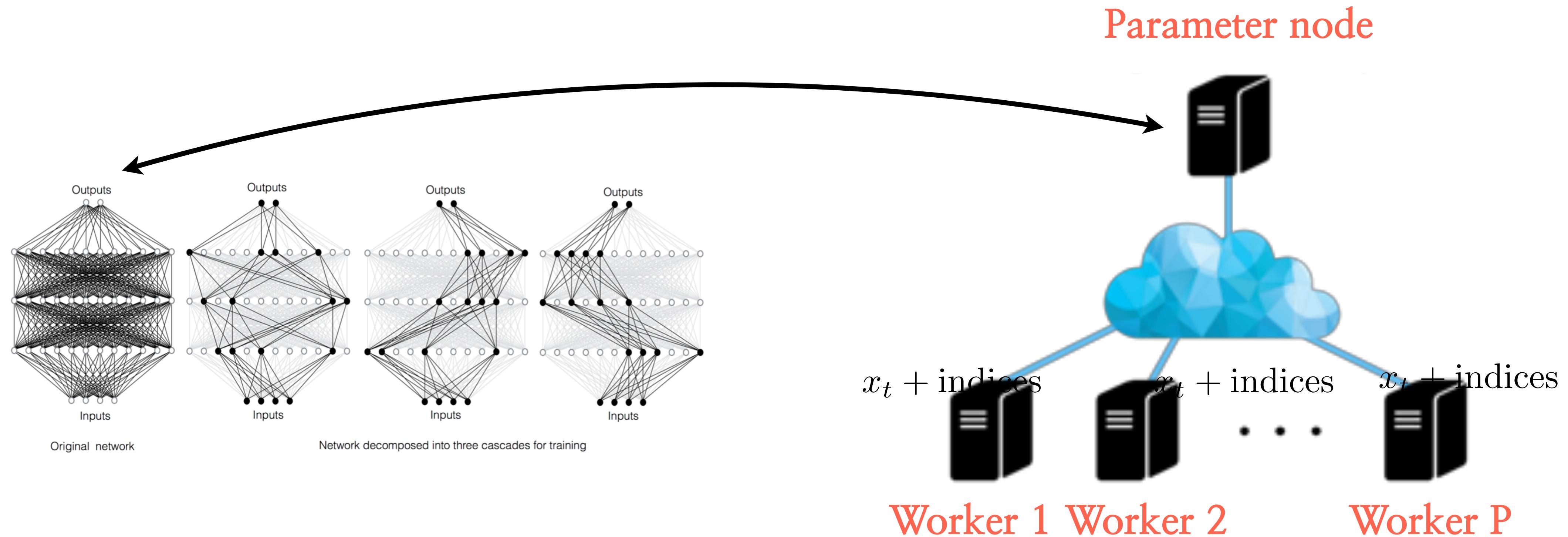
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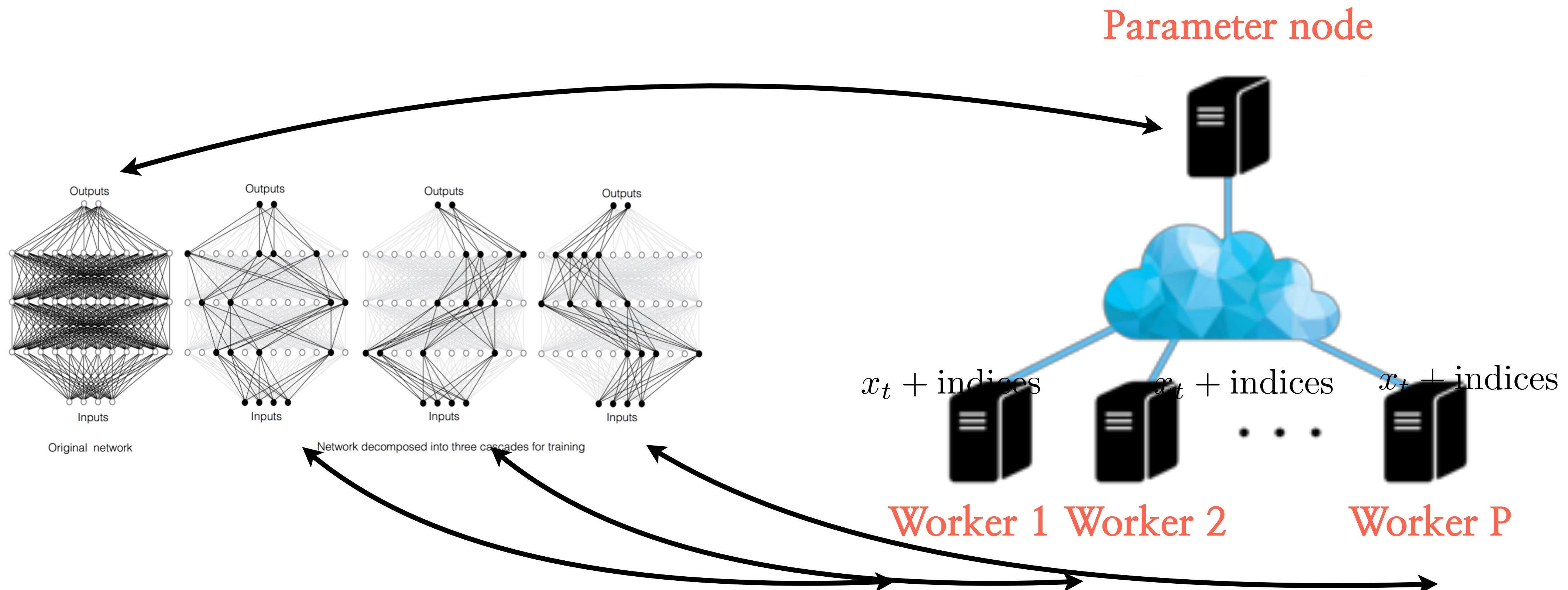
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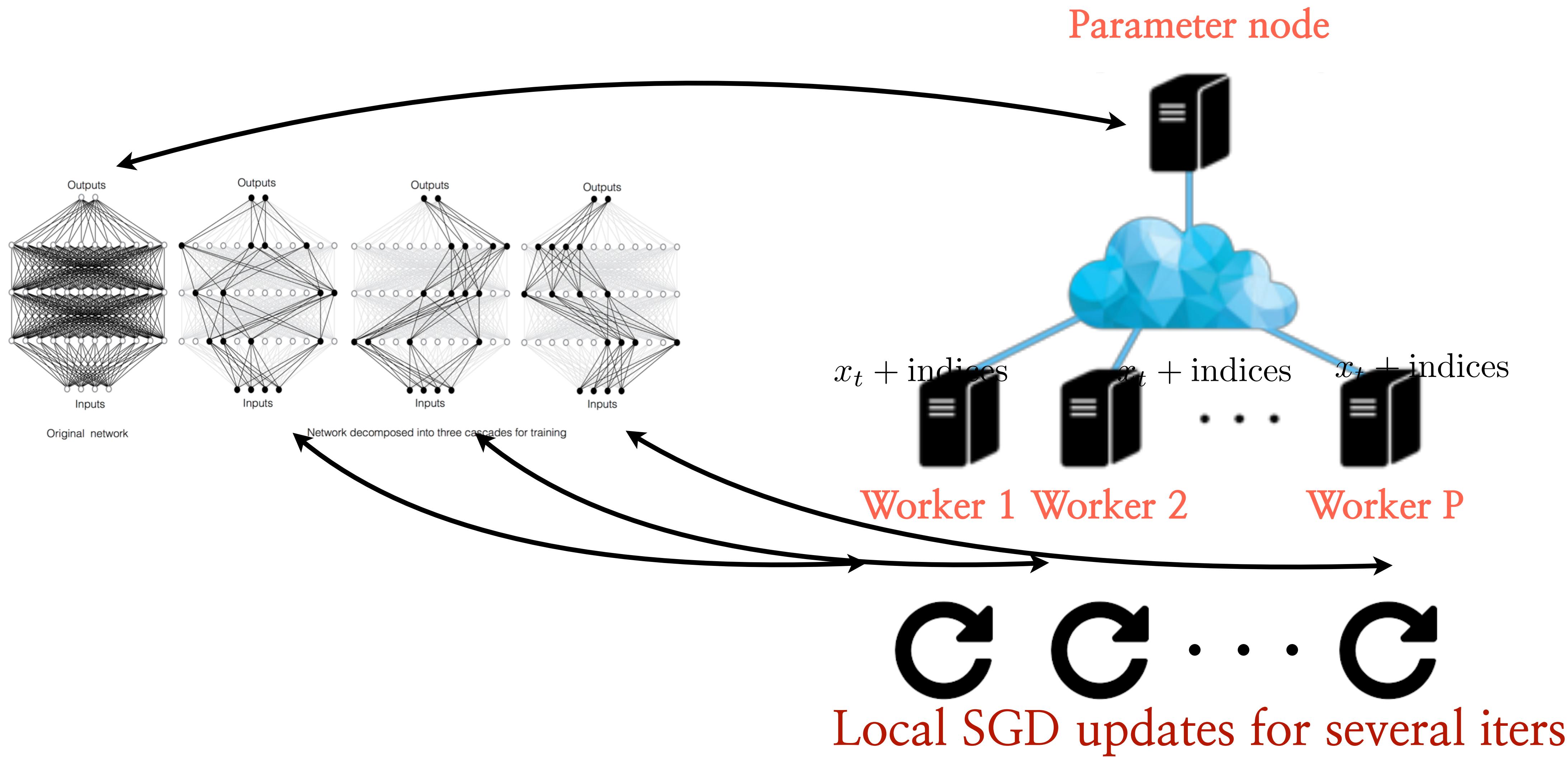
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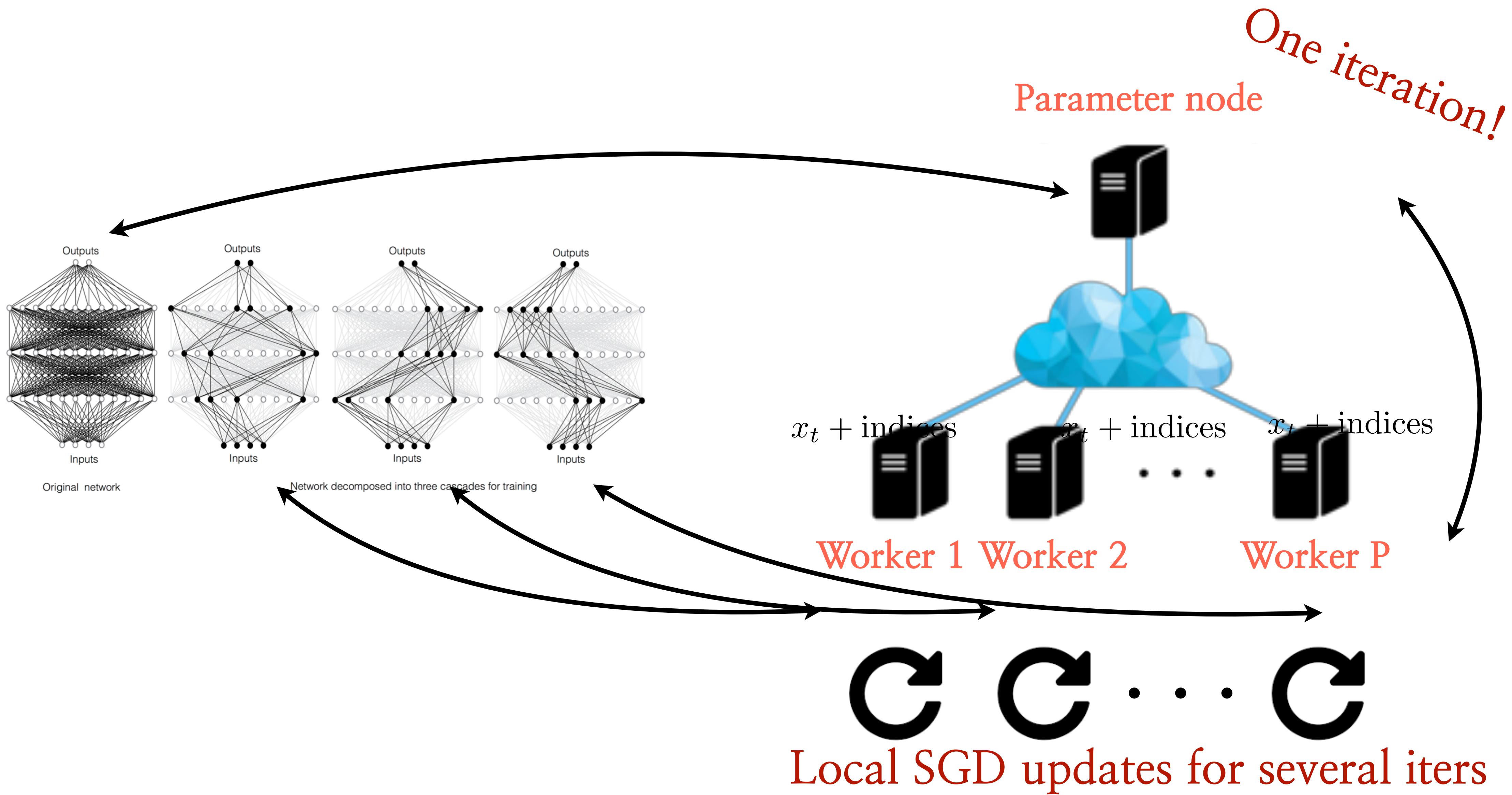
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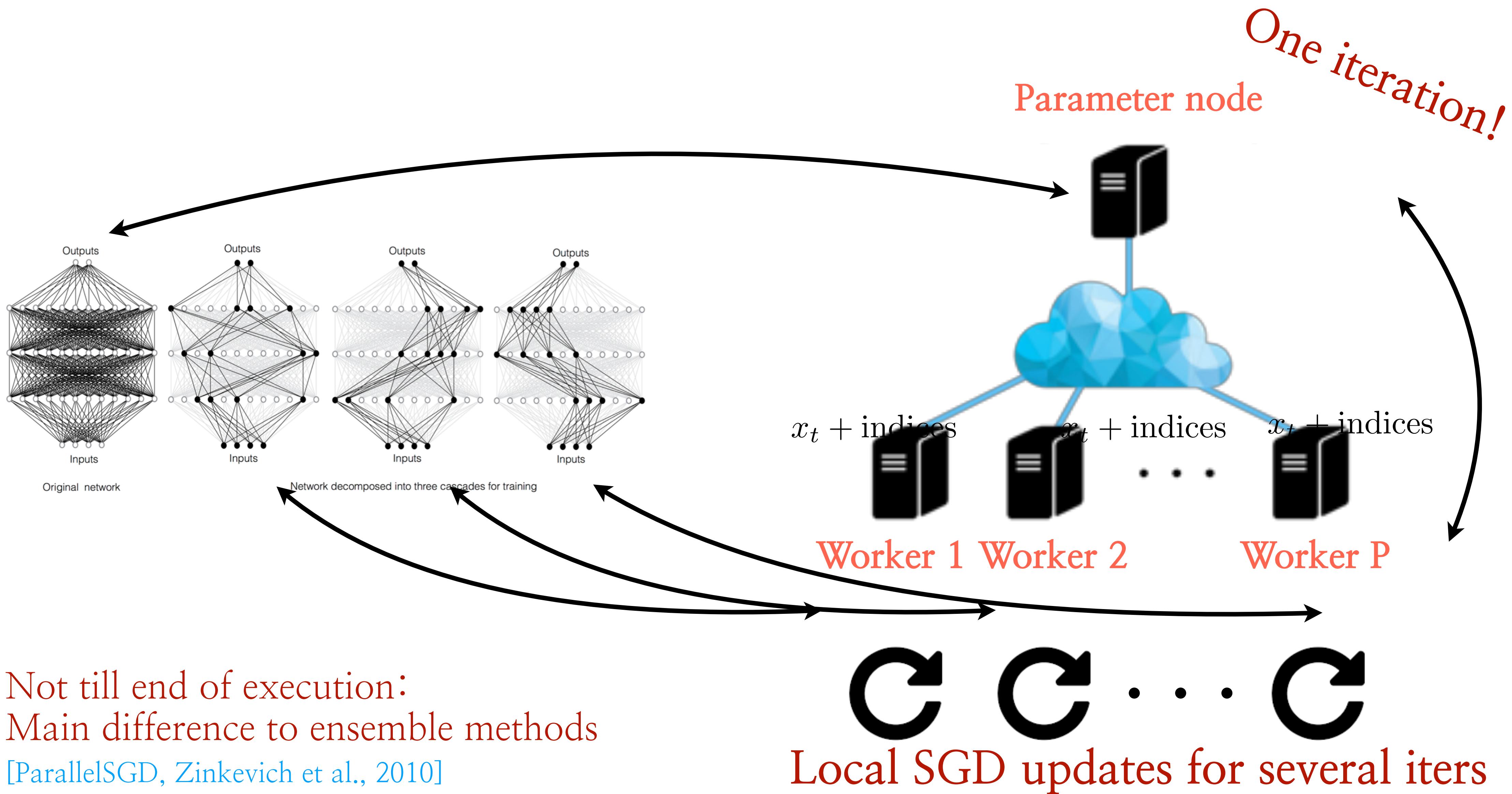
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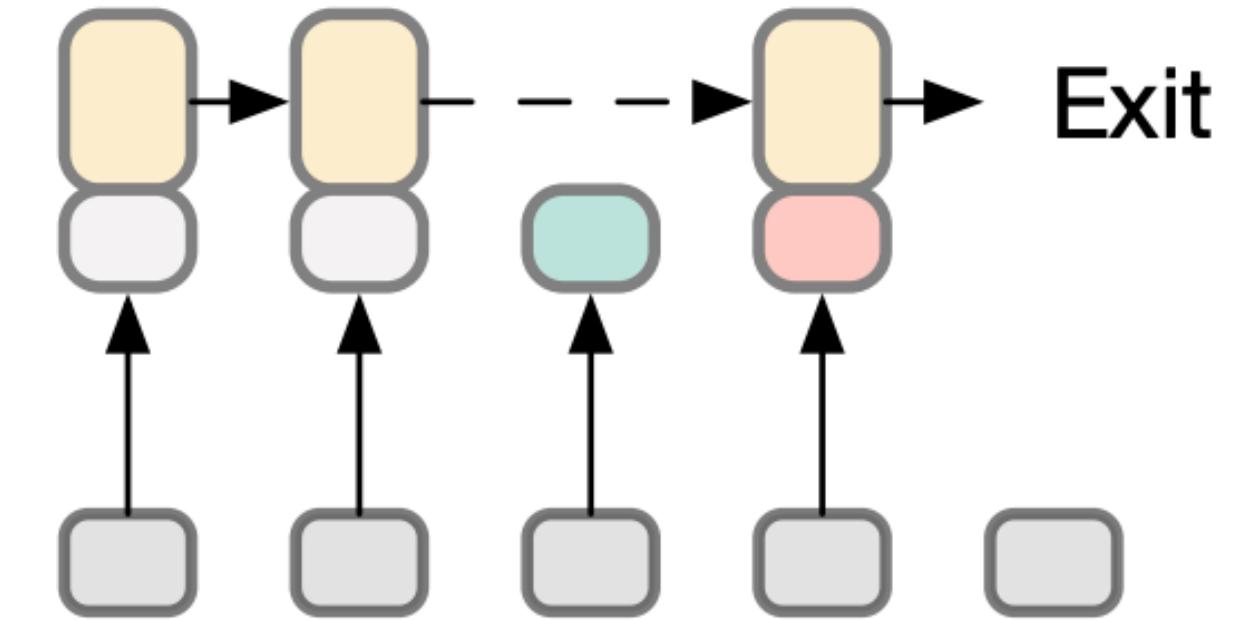


# Independent Subnet Training: NN distr. training



# Algorithms: Dynamic/Approximate Training

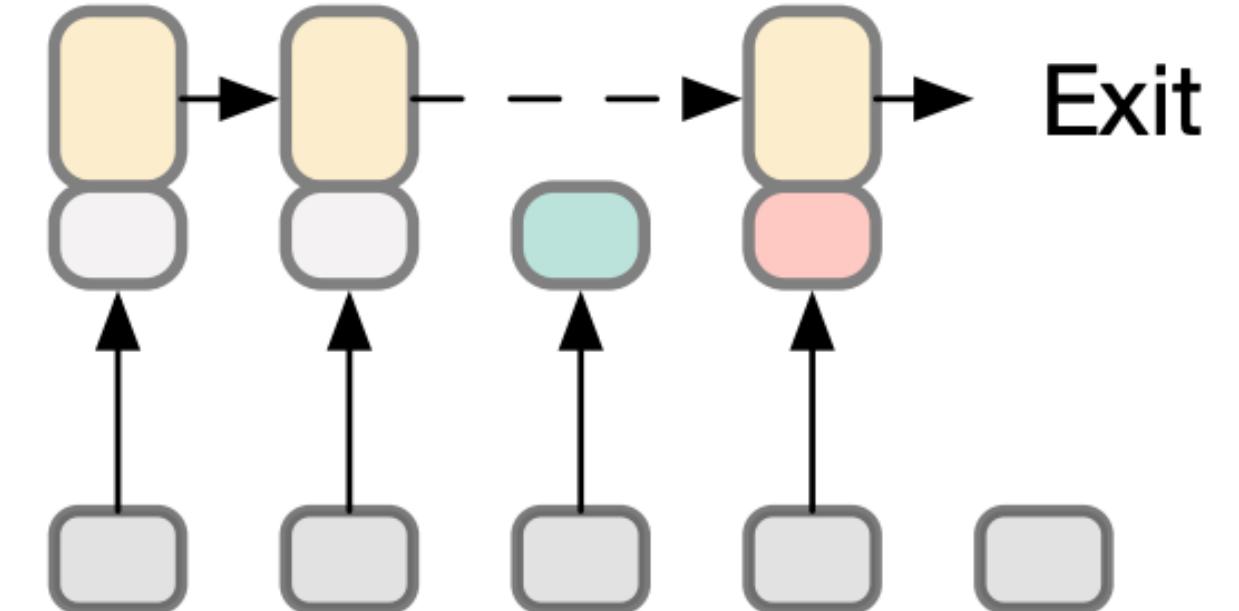
- Skimming: dynamically allocating computation to different time steps, based on the input tokens  
[Huang et al., 2016; Yu et al., 2017; Campos et al., 2018; Li et al., 2019]



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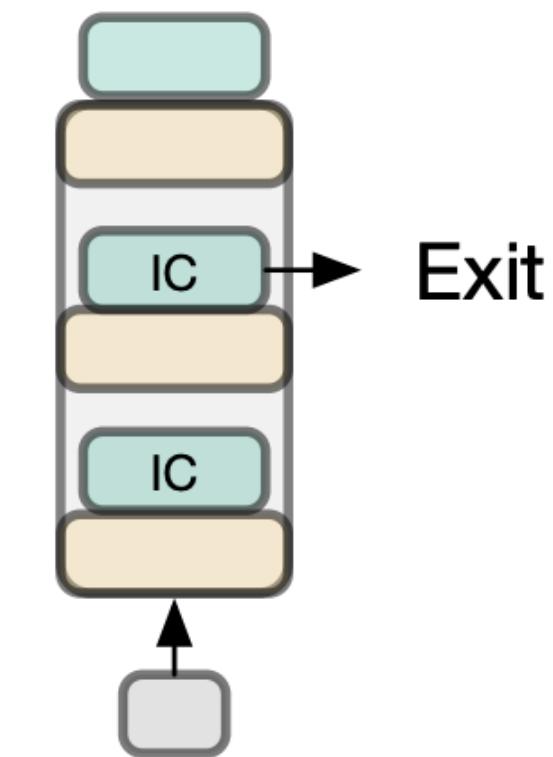
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- Early exit/local objectives per layer

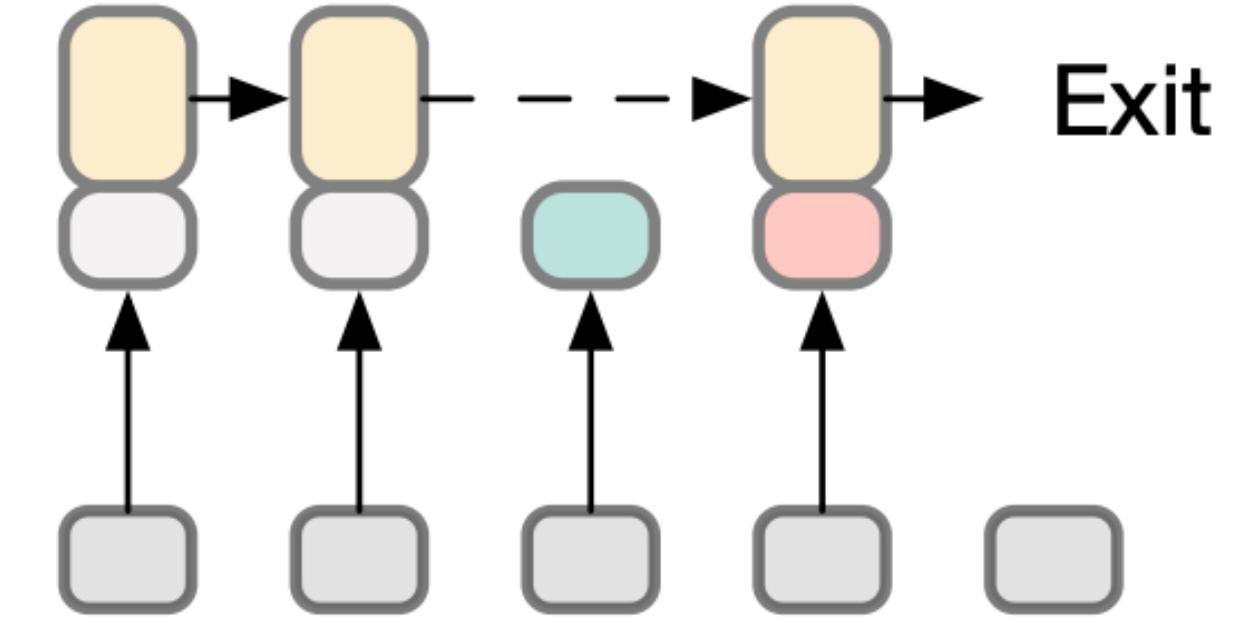
[Kaya et al., 2019; Zhou et al., 2020; Xin et al., 2020, 2021; Sun et al., 2021]



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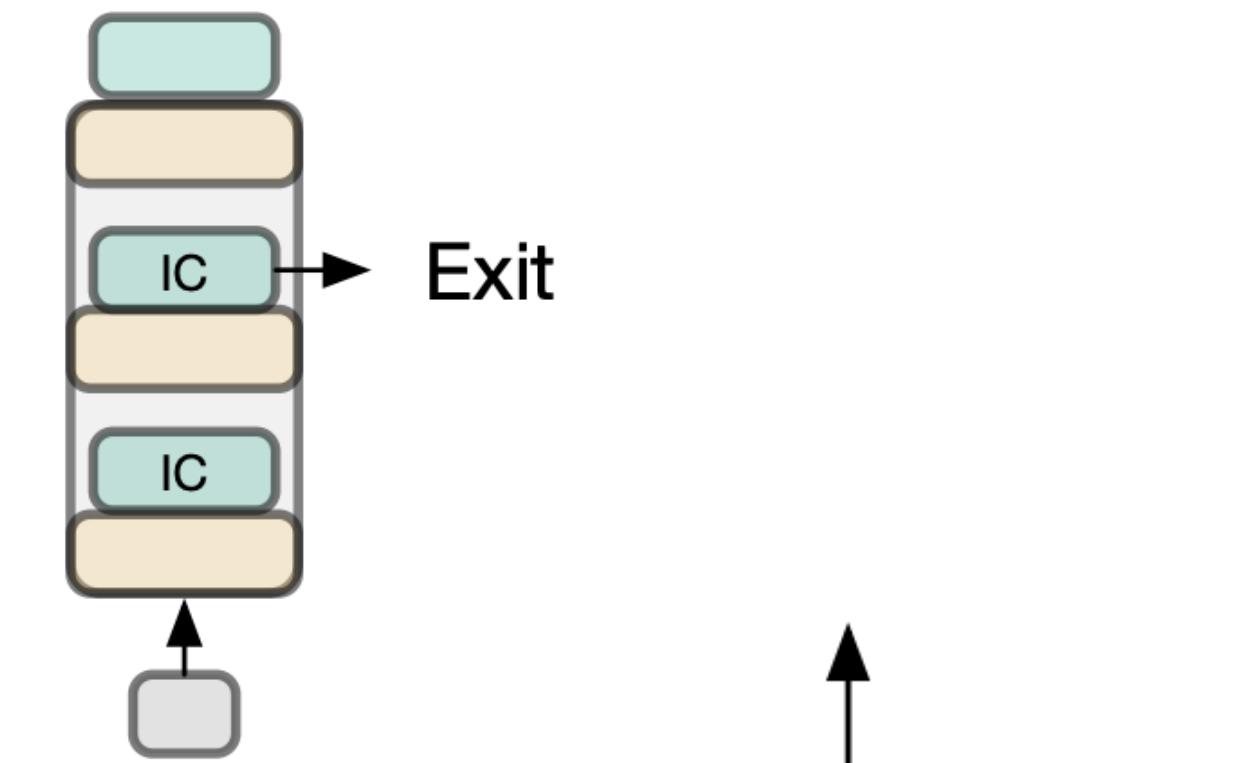
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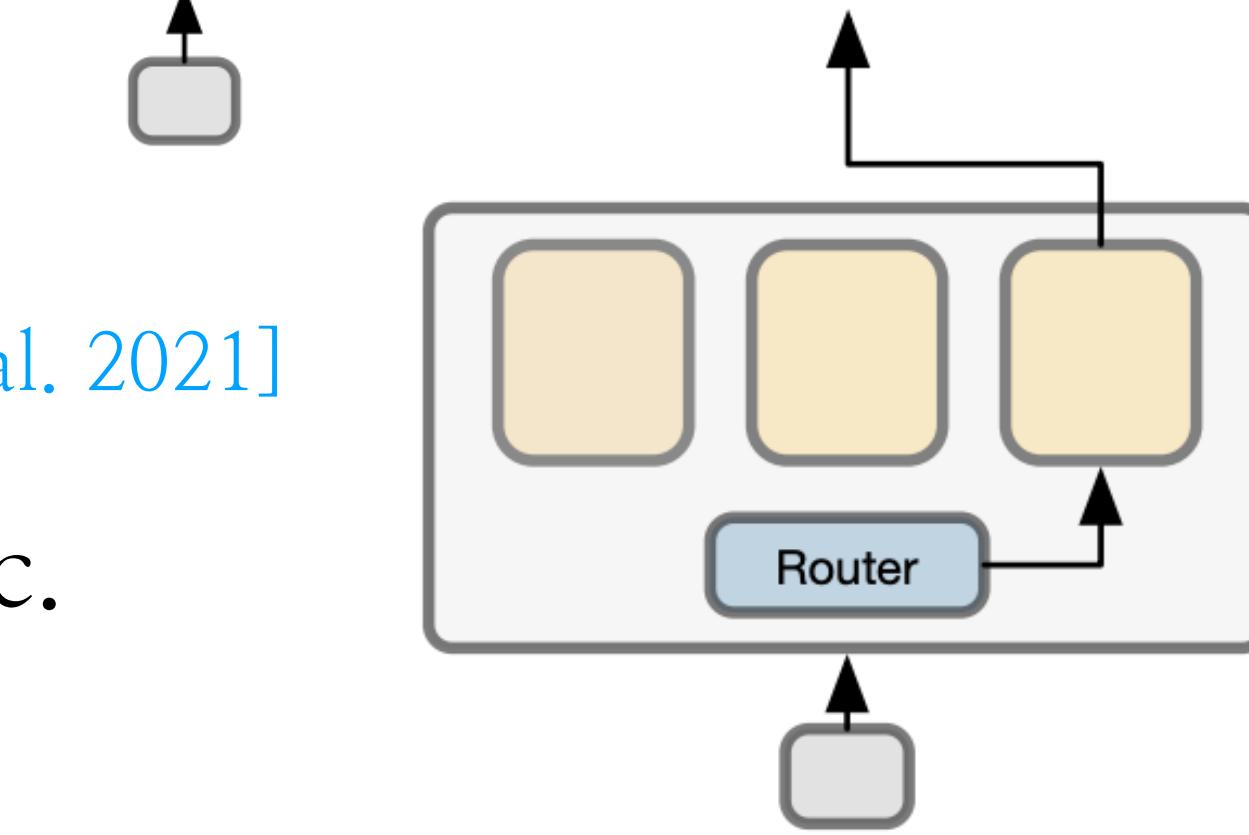
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- Sparsely activated MoEs: a layer typically contains multiple sub-networks (i.e., “experts”) → Structured Dropout.

[Jacobs et al., 1991; Shazeer et al., 2017; Lepikhin et al., 2021; Fedus et al., 2021, Lewis et al. 2021]

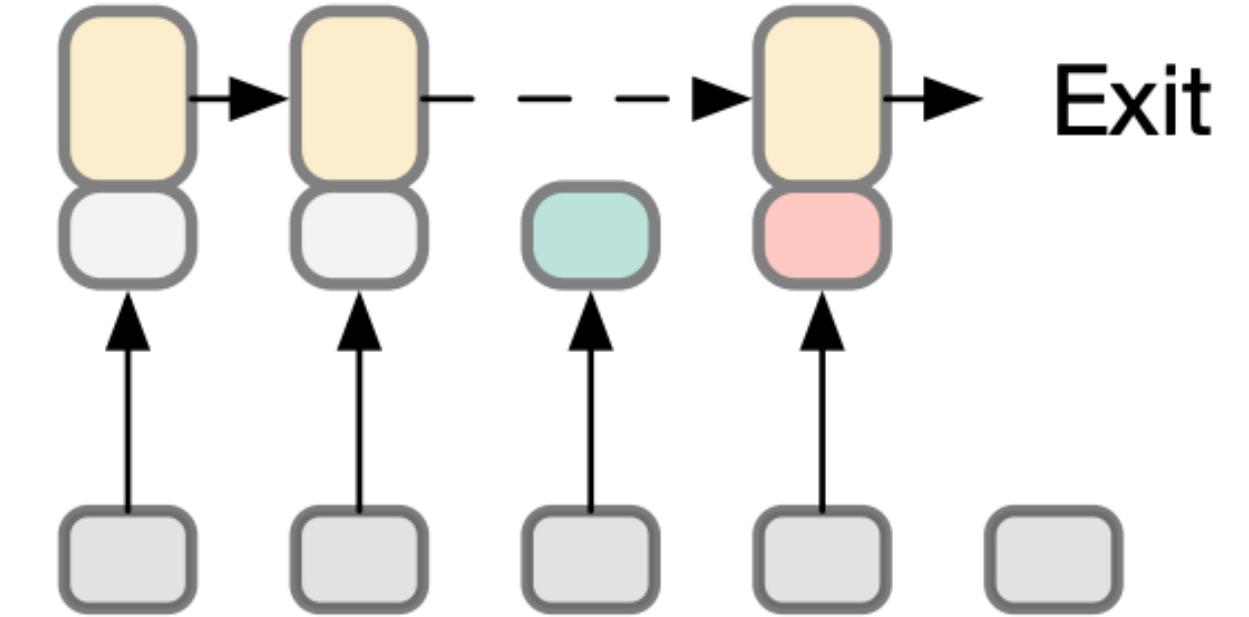


- Examples: Sparsely, GShard, Switch, BASE, DTS, Hash, THOR, etc.

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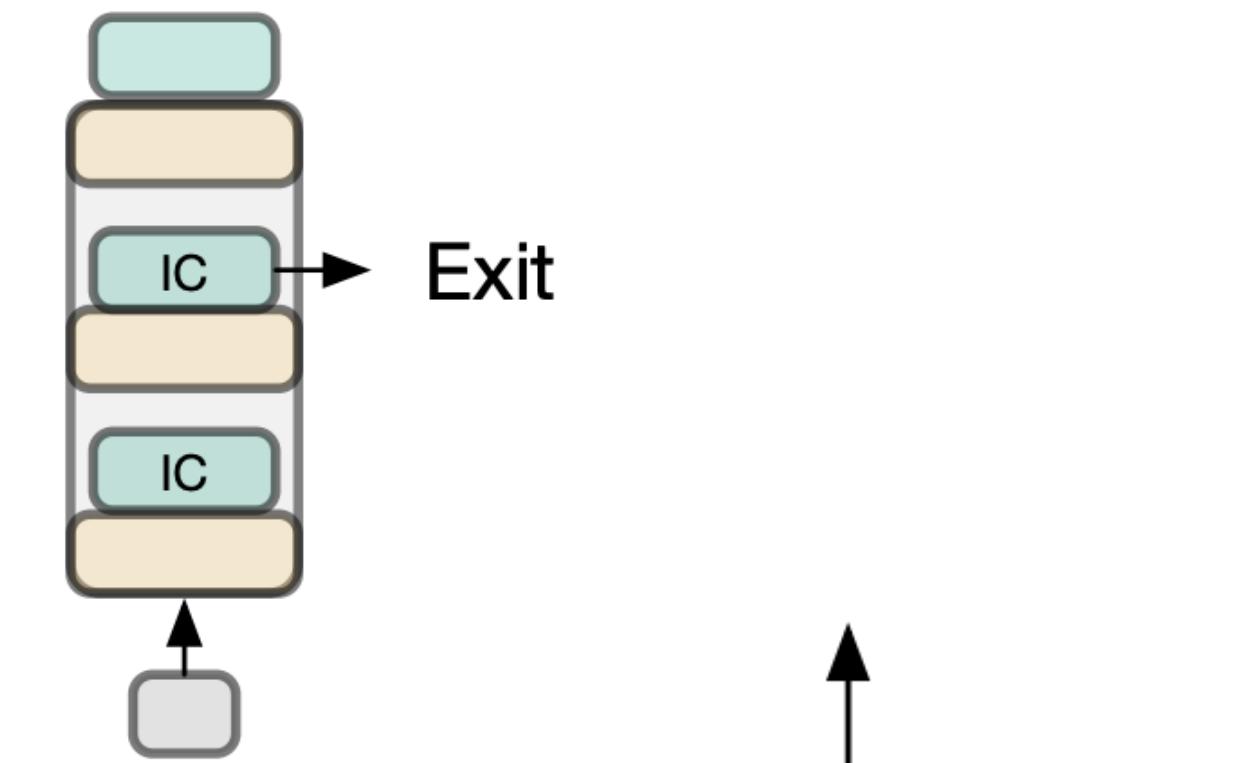
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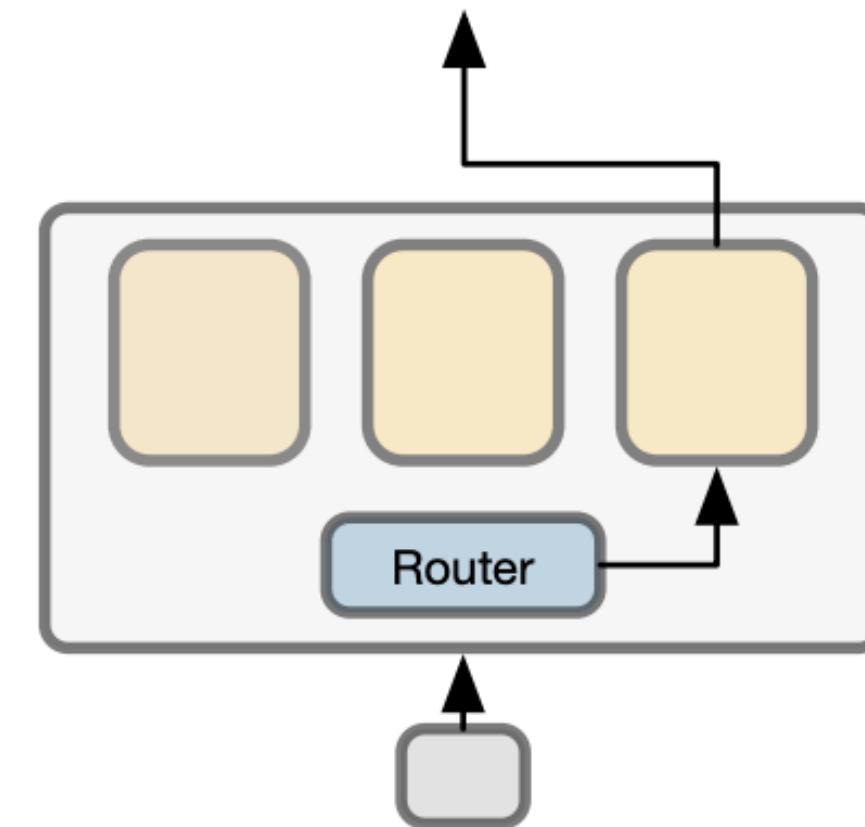


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- Examples: Sparsely, GShard, Switch, BASE, DTS, Hash, THOR, etc.

- IST: unstructured distributed dropout



# Systems: Dynamic/Approximate Distributed Training

- Classical approaches: data parallel, model parallel

[Zinkevich et al., 2010; Stich, 2019; Huang et al. 2019]

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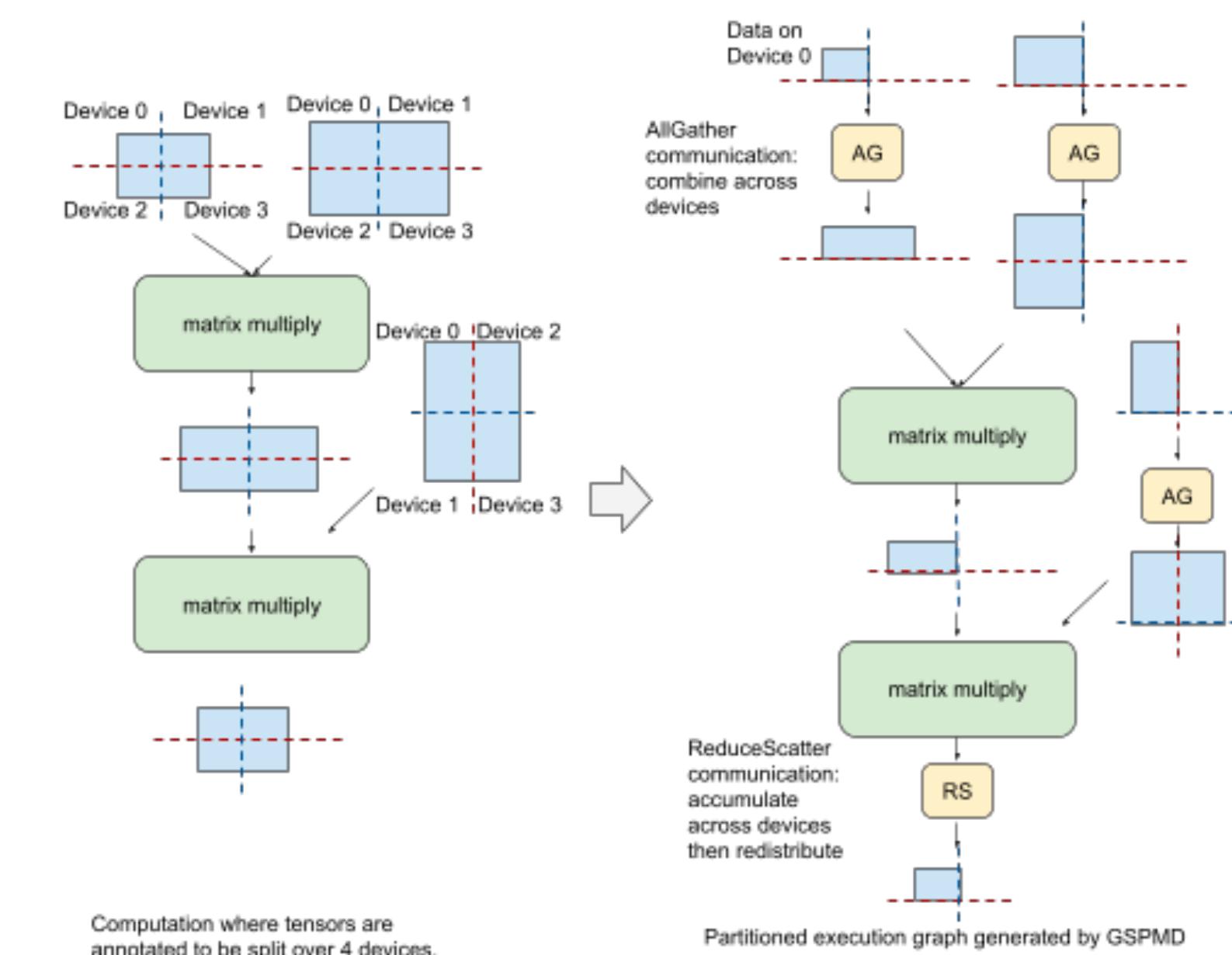
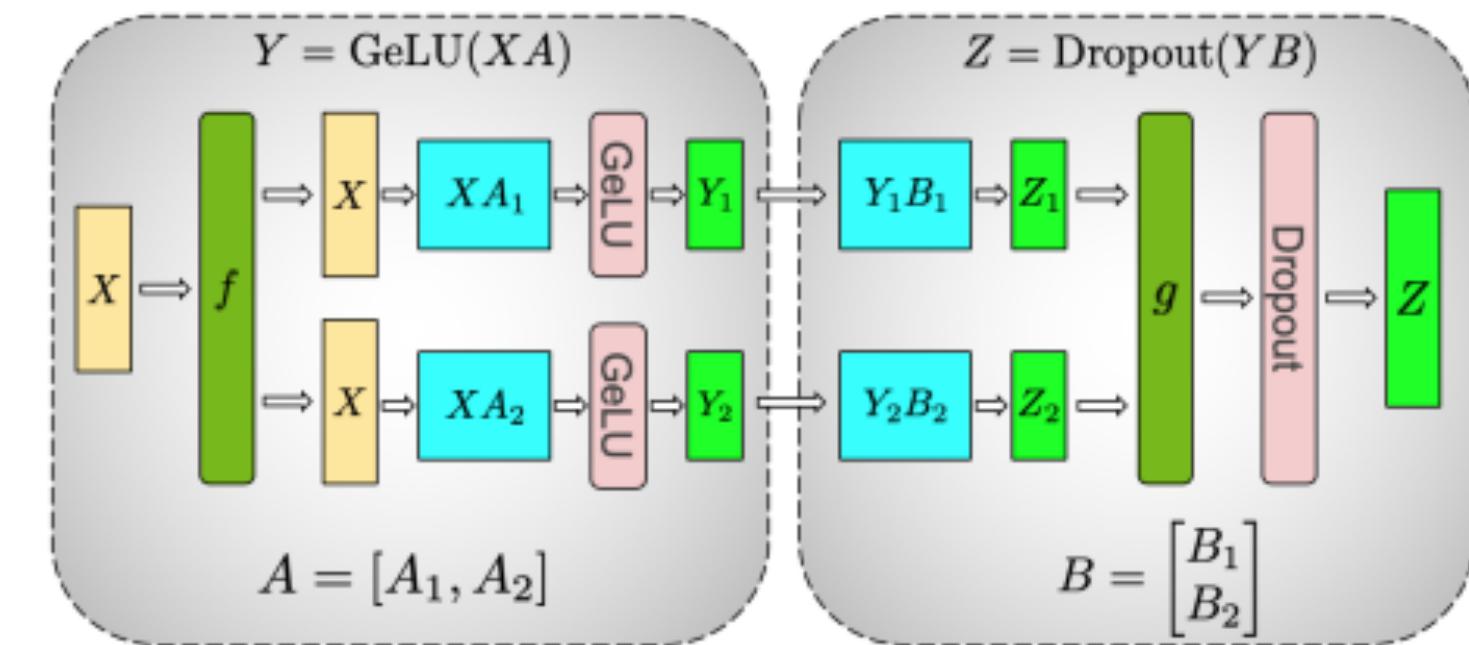
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  - Amazon SageMaker Model Parallelism
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  - Google's GSPMD

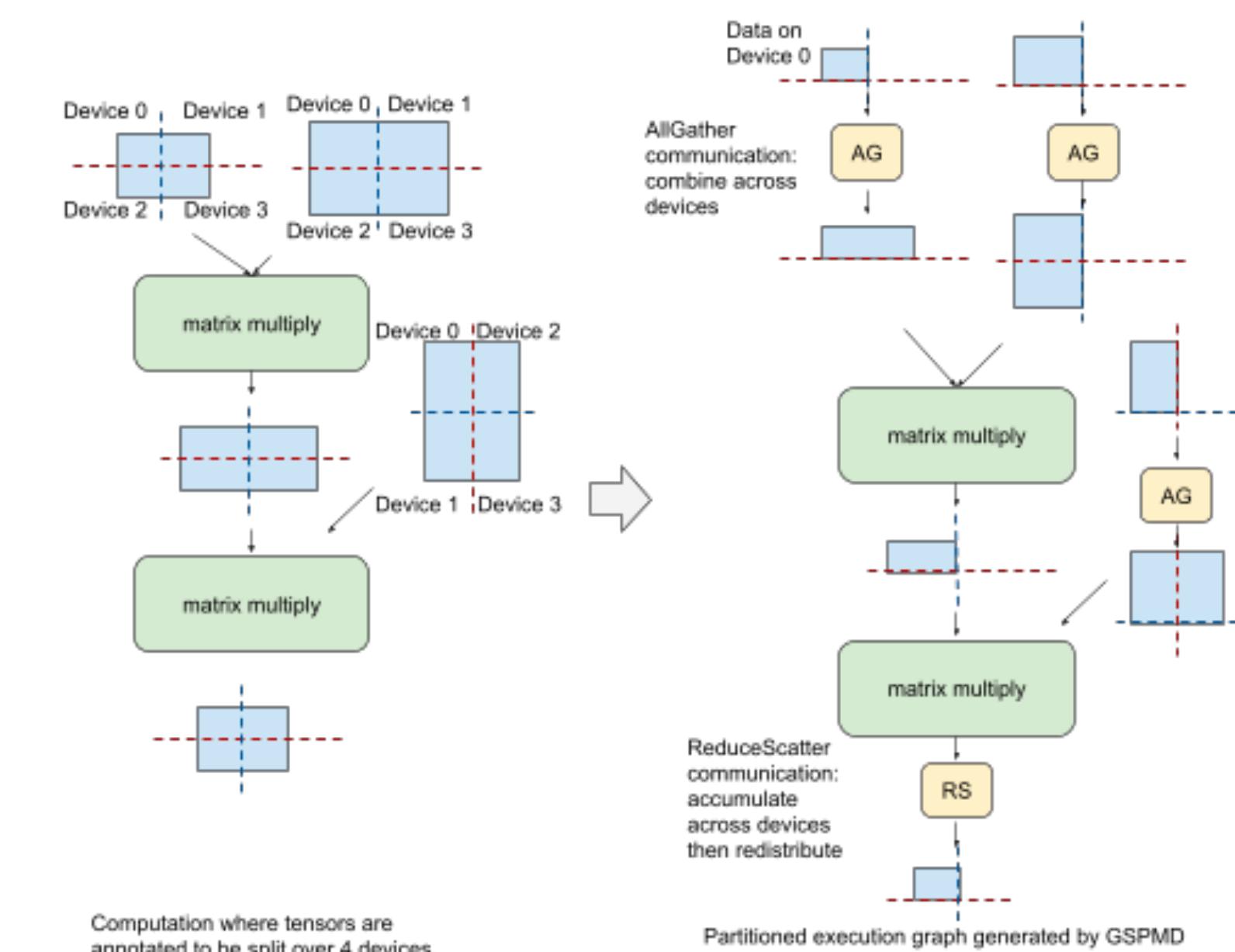
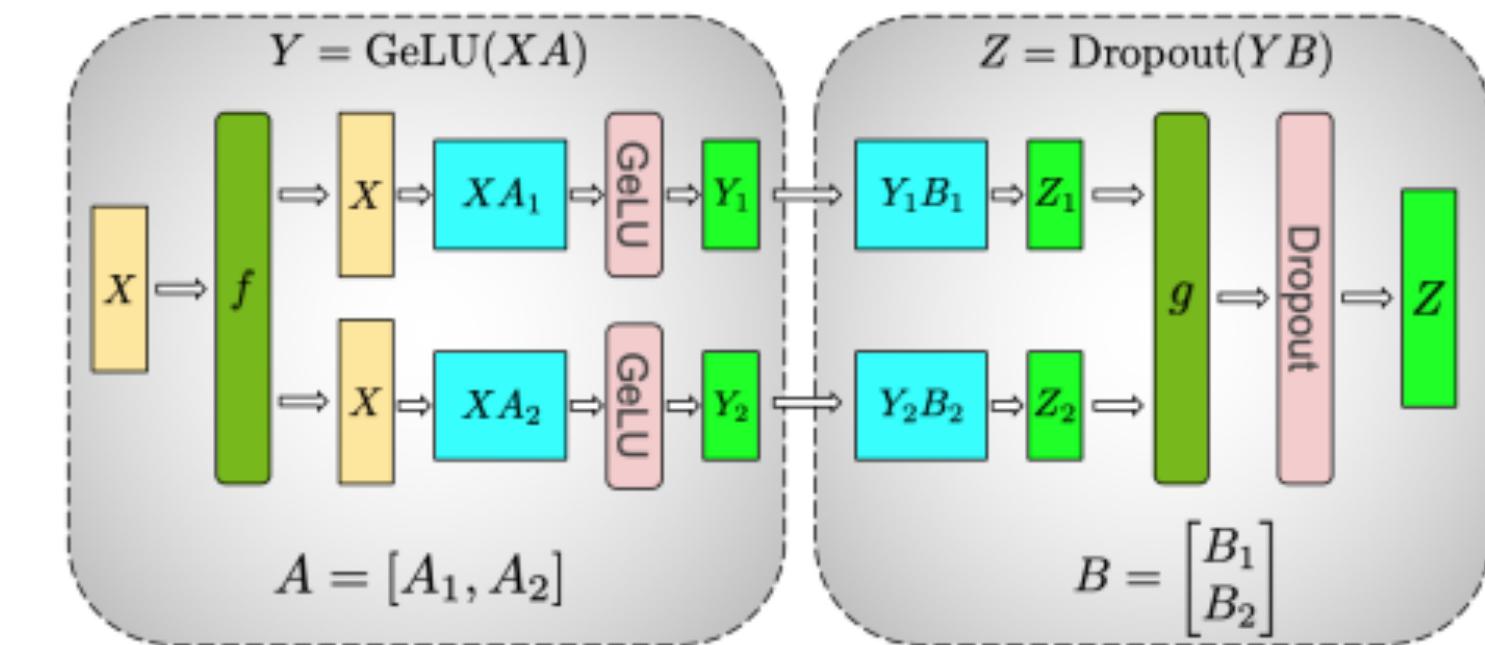


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- IST: End-to-end approximate model tensor/3D parallelism



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## ResIST: Layer-Wise Decomposition of ResNets for Distributed Training

Chen Dun <sup>\* 1</sup> Cameron R. Wolfe <sup>\* 1</sup> Chris Jermaine <sup>1</sup> Anastasios Kyrillidis <sup>1</sup>

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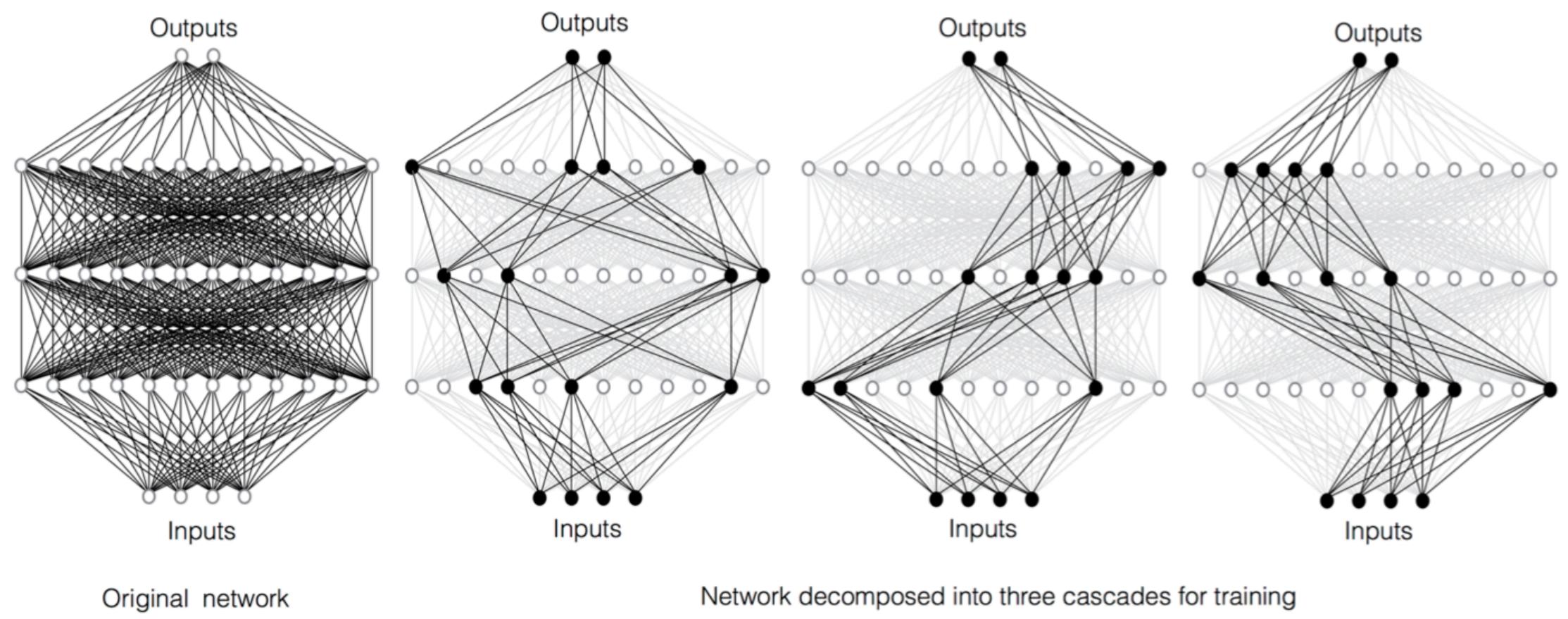
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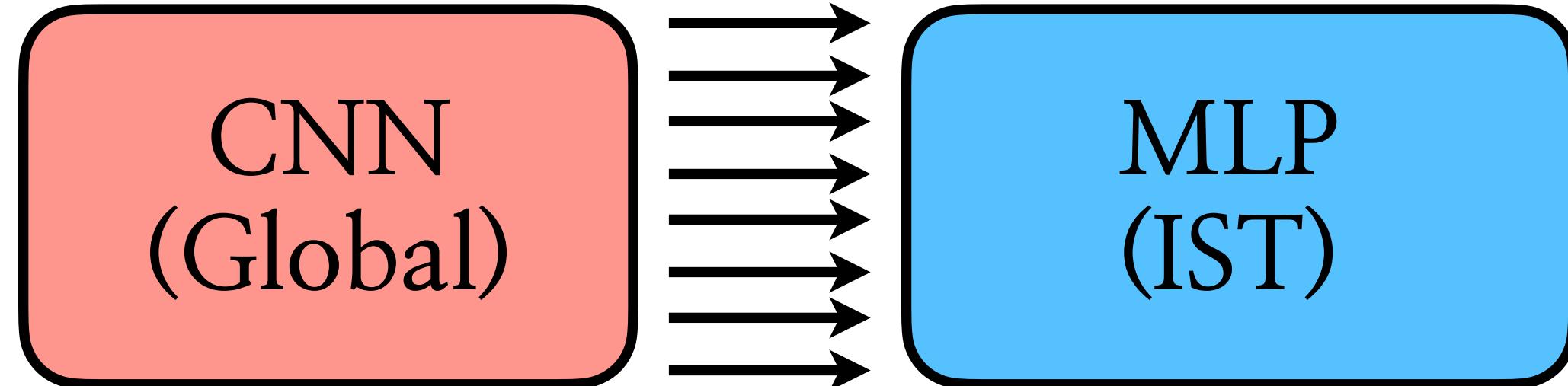
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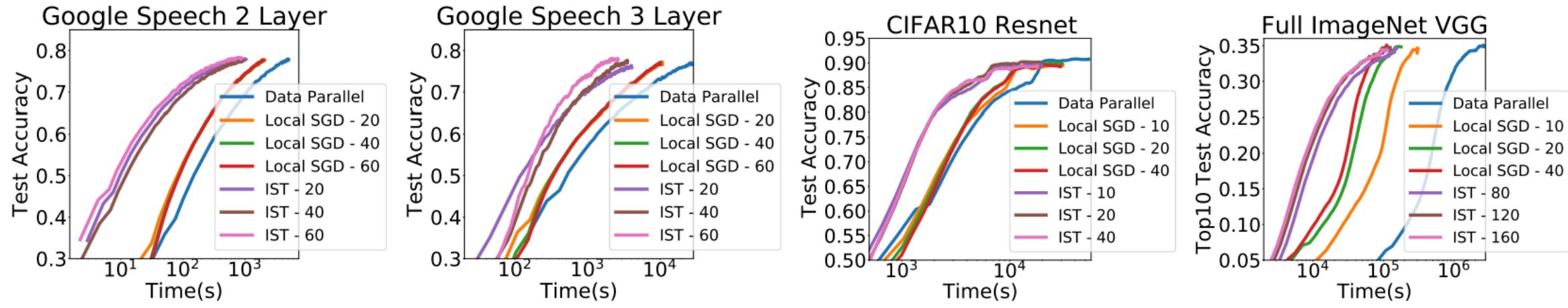
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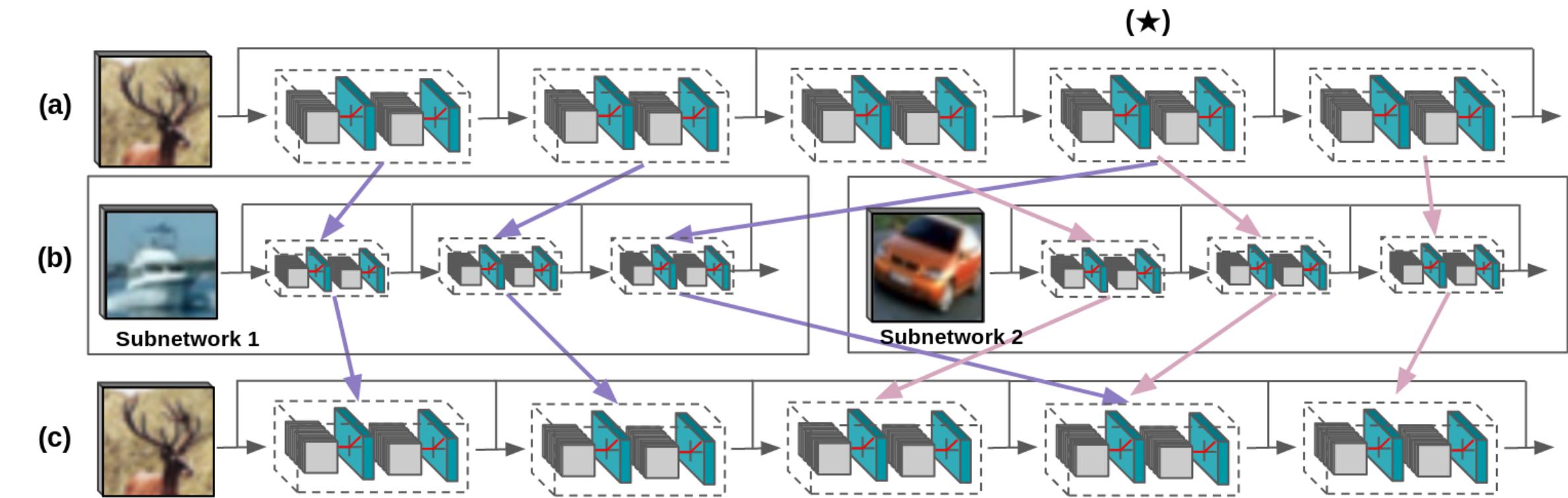
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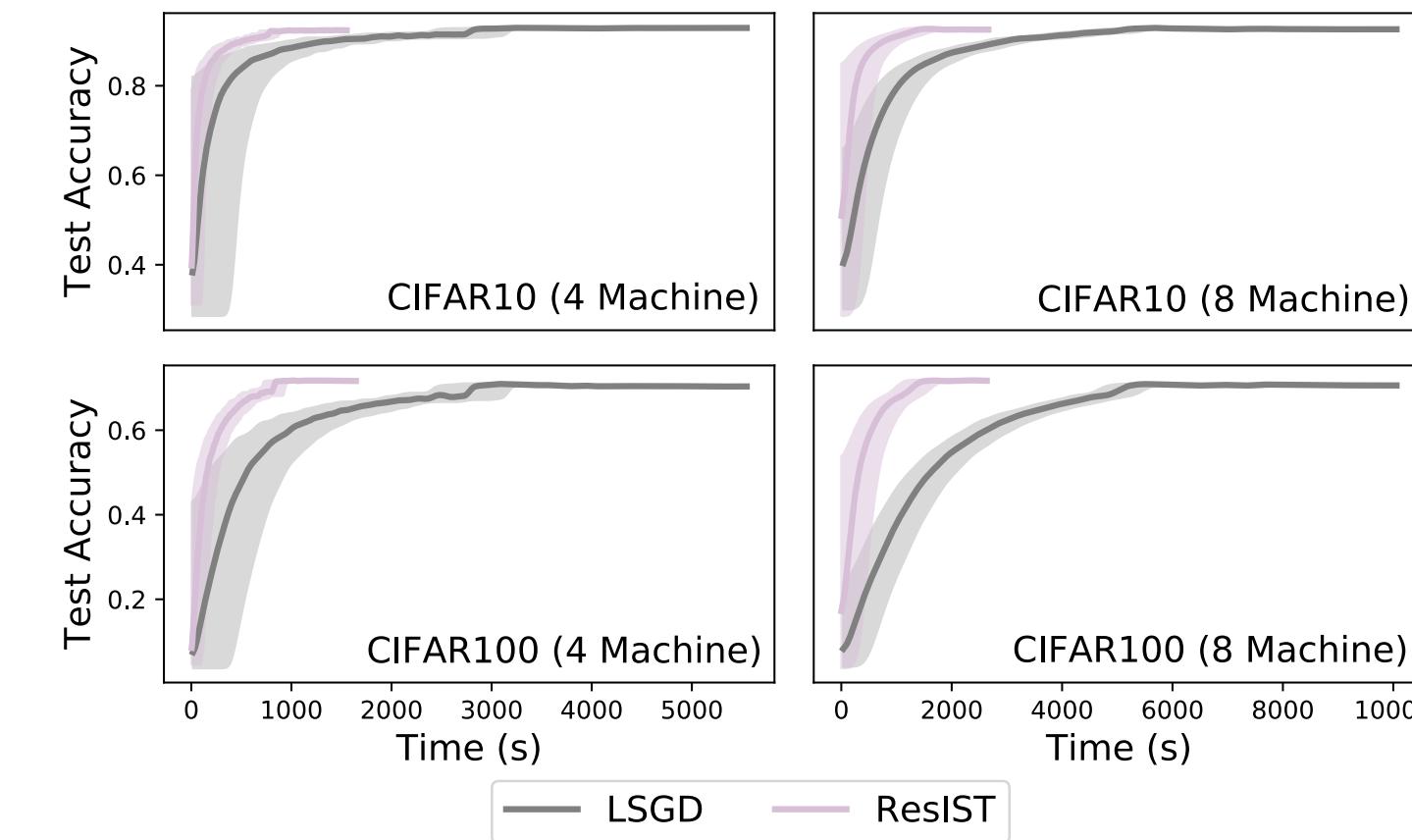
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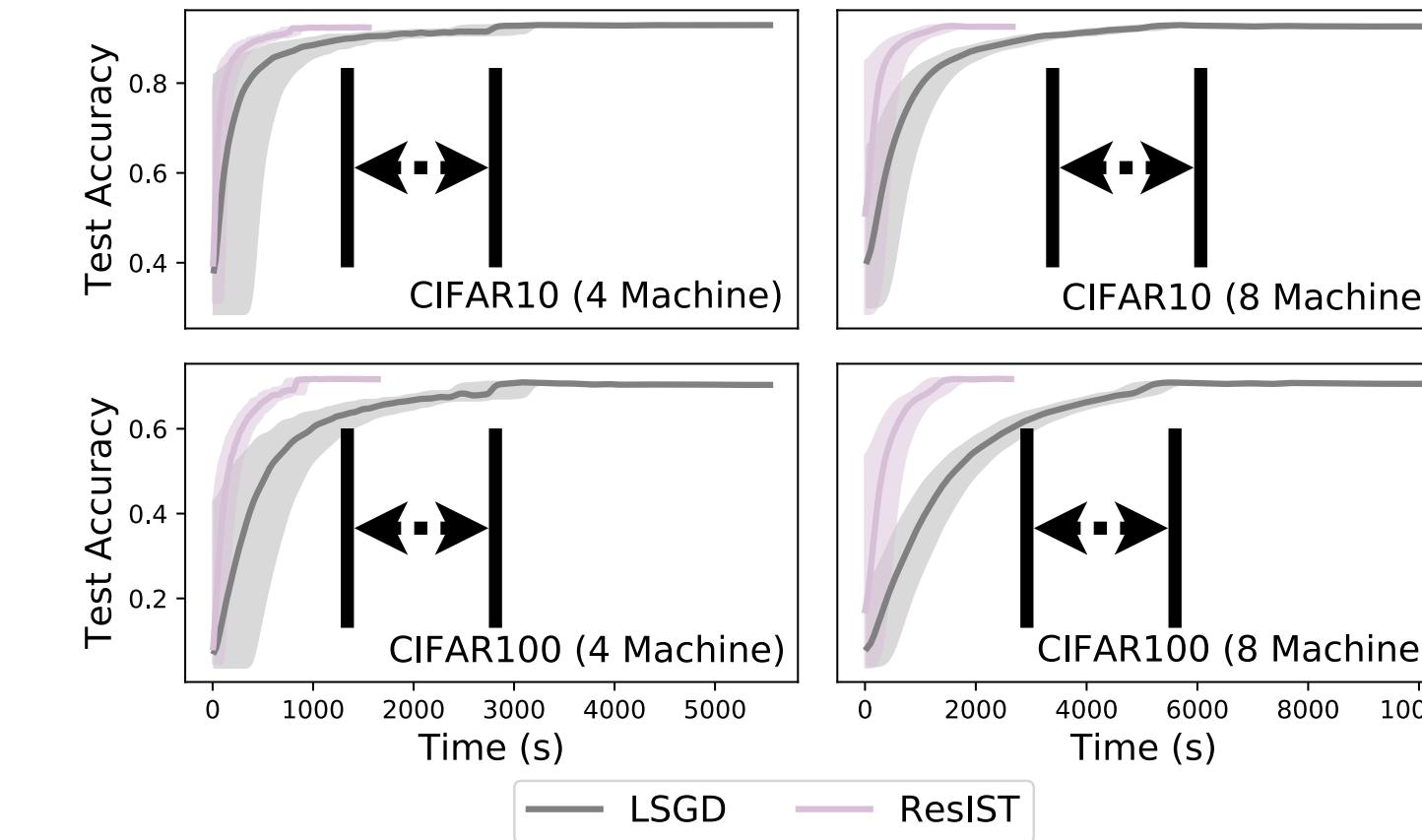
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Acceleration (wall clock) while pertaining accuracy!



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## ResIST: Layer-Wise Decomposition of ResNets for Distributed Training

Chen Dun <sup>\* 1</sup> Cameron R. Wolfe <sup>\* 1</sup> Chris Jermaine <sup>1</sup> Anastasios Kyrillidis <sup>1</sup>

<https://arxiv.org/pdf/2107.00961.pdf>

## GIST: Distributed Training for Large-Scale Graph Convolutional Networks

Cameron Wolfe <sup>\* 1</sup> Jingkang Yang <sup>\* 2</sup> Arindam Chowdhury <sup>3</sup> Chen Dun <sup>1</sup> Artun Bayer <sup>3</sup> Santiago Segarra <sup>3</sup>  
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<https://arxiv.org/pdf/2102.10424.pdf>

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## Distributed Learning of Neural Networks using Independent Subnet Training

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What about theory?

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**Corollary 3.** Let assumptions (1), (2), and (3) hold. Fix the number of dropout iterations to  $K$ , the step size to  $\eta = O(\lambda_0/n\tau \max\{n, p\})$ , and let the number of hidden neurons satisfy  $m = \Theta(n^5K/\xi\theta\lambda_0^4\delta)$ . Then the IST algorithm on a two-layer ReLU neural network converges with probability at least  $1 - \delta$ , according to:

$$\mathbb{E}_{[\mathbf{M}_{k-1}]} [\|\mathbf{y} - \mathbf{u}_k\|_2^2] \leq \left(1 - \frac{1}{4}\eta\theta\tau\lambda_0\right)^k \|\mathbf{y} - \mathbf{u}_0\|_2^2 + O\left(\frac{(1-\xi)^2}{nK} + \frac{\theta-\xi^2}{p} + \left(1 - \frac{1}{\tau}\right)\theta^2(1-\xi)\right)$$

**Theorem 4.** Let assumptions (1) and (4) hold. Then  $\lambda_0 > 0$ . Moreover, let  $\lambda_{\max}$  denote the maximum eigenvalue of  $\mathbf{H}^\infty$ . Fix the number of global iterations to  $K$  and the number of local iterations to  $\tau$ . Let the number of hidden neurons be  $m = \Omega\left(\frac{n^5\tau^2K\lambda_{\max}}{\lambda_0^6\delta}\right)$ , and choose the initialization scale  $\kappa = \sqrt{n\lambda_{\max}}\lambda_0^{-1}$ . Let  $\gamma = (1 - p^{-1})^{\frac{1}{3}}$ . Then, Algorithm (1) with a constant step-size  $\eta = O\left(\frac{\lambda_0}{n^2} \min\left\{\frac{p}{\gamma^2\tau}, 1\right\}\right)$  converges with probability at least  $1 - \delta$ , according to:

$$\mathbb{E}_{[\mathbf{M}_{k-1}]} [\|\mathbf{y} - \mathbf{u}_k\|_2^2] \leq \left(\gamma + (1 - \gamma)\left(1 - \frac{\eta\lambda_0}{2}\right)^\tau\right)^k \|\mathbf{y} - \mathbf{u}_0\|_2^2 + O\left(\frac{\gamma\tau n\kappa^2\lambda_{\max}}{\lambda_0^2}\right).$$

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**Theorem B.1** (Convergence Rate of Gradient Descent for ResIST). Assume there are  $S$  workers,  $\ell$  local and  $T$  global steps. Assume the depth of the whole ResNet is  $H$ . Assume for all data indices  $i \in [n]$ , the data input satisfies  $\|\mathbf{x}_i\|_2 = 1$ , the data output satisfies  $|y_i| = O(1)$ , and the number of hidden nodes per layer satisfies  $m =$

$$\Omega\left(\max\left\{\frac{n^4}{\lambda_{\min}^4(\mathbf{K}^{(H)})H^6}, \frac{n^2}{\lambda_{\min}^2(\mathbf{K}^{(H)})H^2}, \frac{n}{\delta}, \frac{n^2 \log\left(\frac{Hn}{\delta}\right)}{\lambda_{\min}^2(\mathbf{K}^{(H)})}\right\}\right).$$

Set the step size  $\eta = O\left(\frac{\lambda_{\min}(\mathbf{K}^{(H)})H^2}{n^2\ell^2S}\right)$  in gradient descent in local training iteration, and follow the procedure as in

Algorithm 1. Let the squared-norm loss be  $L(\theta(t)) := \frac{1}{2}\|\mathbf{y} - f(\theta(t))\|_2^2$ , per  $t$  global synchronization round,  $t = 1, 2, \dots, T$ ; here,  $\mathbf{y}$  corresponds to the data “labels”, and  $\theta(t)$  and  $f(\theta(t))$  represent the parameters and the output of the whole ResNet, respectively, after  $t$ -global rounds of ResIST. Here,  $\theta$  includes weights  $\mathbf{W}^{(h)}$  at depth  $h$  and the last layer’s weights  $\mathbf{a}$ . Then, with probability at least  $1 - \delta$  over the random initialization, we have:

$$L(\theta(t)) \leq \left(1 - \frac{\eta\ell\lambda_{\min}(\mathbf{K}^{(H)})}{2}\right)^t \cdot L(\theta(0)).$$

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## ON THE CONVERGENCE OF SHALLOW NEURAL NETWORK TRAINING WITH RANDOMLY MASKED NEURONS

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What about more architectures?

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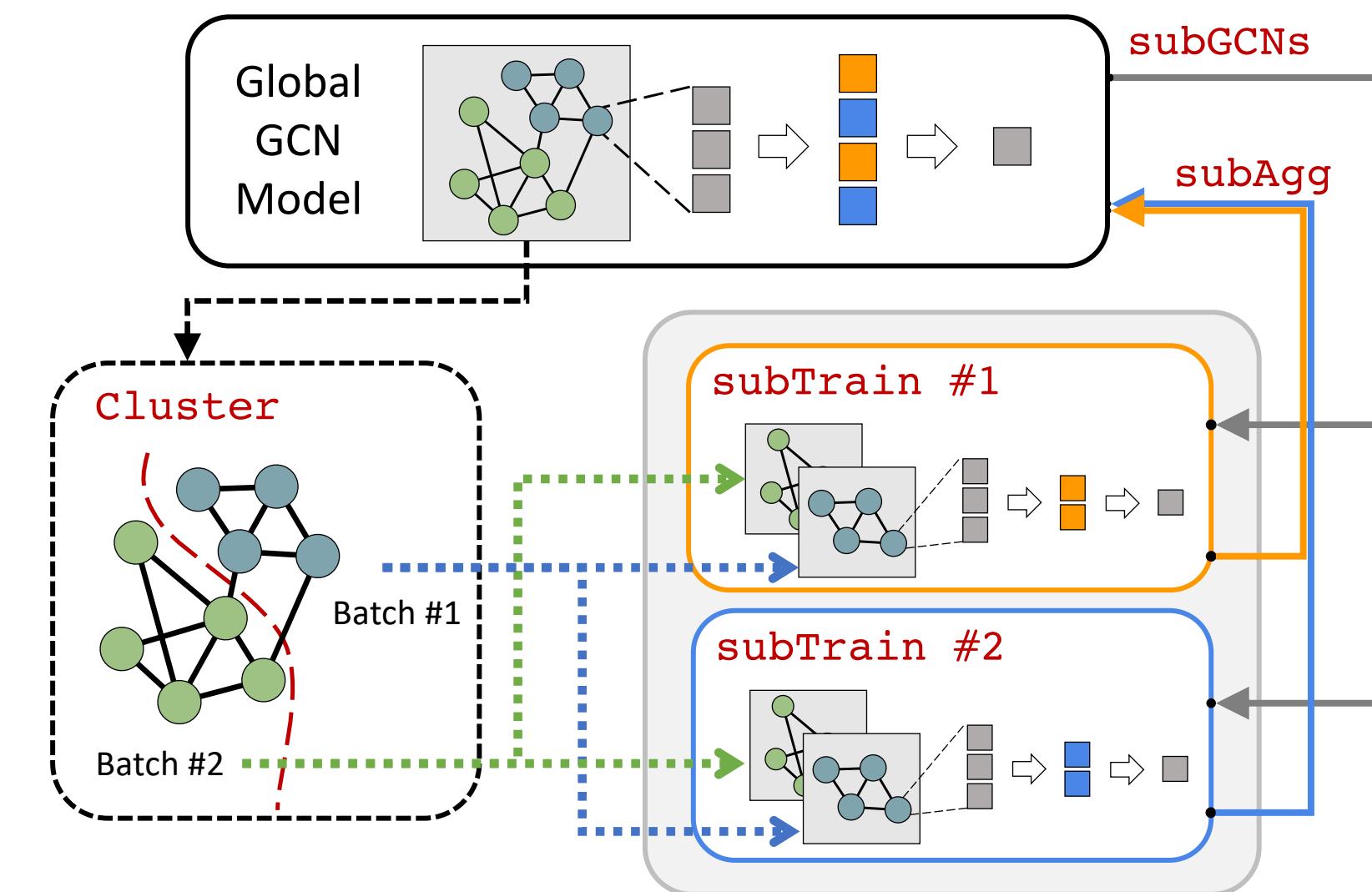
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$L$	$m$	F1 Score (Time in hours)				
		$d_i = 400$	$d_i = 4\,096$	$d_i = 8\,192$	$d_i = 16\,384$	$d_i = 32\,768$
2	-	89.38 (1.81)	90.58 (5.17)	OOM	OOM	OOM
	2	87.48 (1.25)	90.09 (1.70)	90.87 (2.76)	90.94 (9.31)	90.91 (32.31)
	4	84.82 (1.11)	88.79 (1.13)	89.76 (1.49)	90.10 (2.24)	90.17 (5.16)
	8	82.56 (1.13)	87.16 (1.11)	88.31 (1.20)	88.89 (1.39)	89.46 (1.76)
3	-	89.73 (2.32)	90.99 (9.52)	OOM	OOM	OOM
	2	87.79 (1.56)	90.40 (2.12)	90.91 (4.87)	91.05 (17.7)	OOM
	4	85.30 (1.37)	88.51 (1.42)	89.75 (2.07)	90.15 (3.44)	OOM
	8	82.84 (1.37)	86.12 (1.34)	88.38 (1.37)	88.67 (1.88)	88.66 (2.56)
4	-	89.77 (3.00)	91.02 (14.20)	OOM	OOM	OOM
	2	87.75 (1.79)	90.36 (2.77)	91.08 (6.92)	91.09 (26.44)	OOM
	4	85.32 (1.58)	88.50 (1.65)	89.76 (2.36)	90.05 (4.93)	OOM
	8	83.45 (1.56)	86.60 (1.55)	88.13 (1.61)	88.44 (2.30)	OOM

**Table 3:** Performance of GraphSAGE models of different widths trained with GIST on Amazon2M.  $m = “-”$  refers to the baseline and “OOM” marks experiments that cause out-of-memory errors. *GIST* enables training of higher-performing, ultra-wide models.

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**Theorem 2.** Suppose assumptions 2-4, and property 2 hold. Moreover, suppose in each global iteration the masks are generated from a categorical distribution with uniform mean  $1/m$ . Fix the number of global iterations to  $T$  and local iterations to  $\zeta$ . If the number of hidden neurons satisfies  $d_1 = \Omega\left(\frac{n^3\zeta^2T^2}{\delta^2\gamma(1-\gamma)^2\lambda_0^4}\left(n + \frac{d}{m^2}\|\bar{\mathbf{A}}^2\|_{1,1}\right)\right)$ , then procedure (3) with constant step size  $\eta = O\left(\frac{\lambda_0}{n^2\|\mathbf{A}^2\|_{1,1}}\right)$  converges according to

$$\mathbb{E}_{[\mathcal{M}_{t-1}], \Theta_0, \mathbf{a}} [\|\mathbf{y} - \hat{\mathbf{y}}(t)\|_2^2] \leq \left(\gamma + (1-\gamma)\left(1 - \frac{\eta\lambda_0}{2}\right)^\zeta\right)^t \mathbb{E}_{\Theta_0, \mathbf{a}} [\|\mathbf{y} - \hat{\mathbf{y}}(0)\|_2^2] + O\left(\frac{(m-1)^2\zeta\|\bar{\mathbf{A}}^2\|_{1,1}nd}{\gamma m^2 d_1}\right)$$

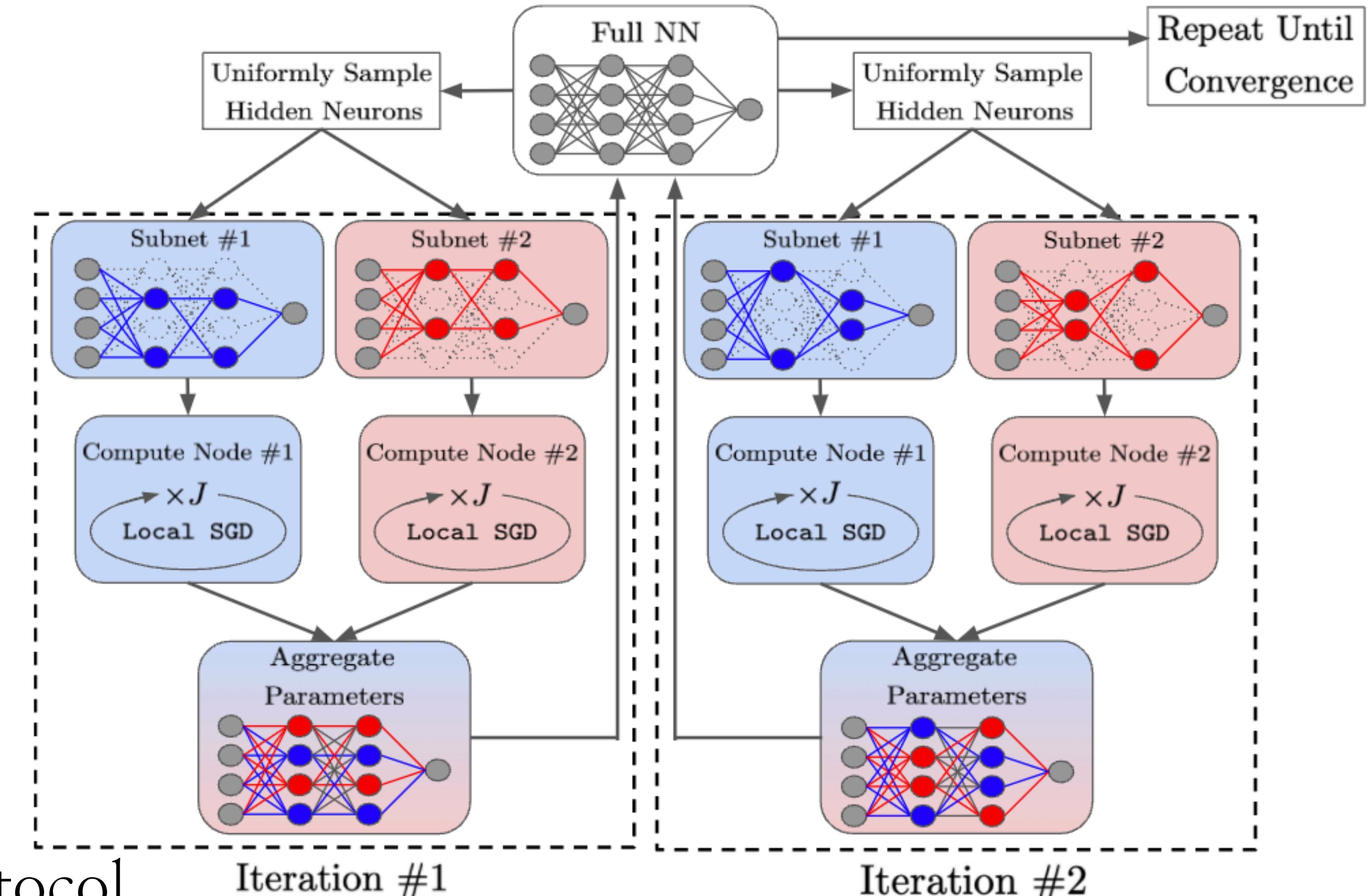
with probability at least  $1 - \delta$ .

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# Landscape of IST-related papers

- Idea of splitting the model into smaller ones (as part of MM) – Google  
[Caldas, Konecny, McMahan, Talwalkar, 2018]  
**Remark:** IST is different since each worker receives a different model to aggregate
- One concurrent work from a FL perspective (to reduce comm/comp)
- Several works after IST:
  - Helios [Xu, Yu, Xiong and Chen, 2019]
  - HeteroFL [Diao, Ding, and Tarokh, 2020]
  - FjORD [Horvath, Laskaridis, Almeida, Leontiadis, Venieris and Lane, 2021]
  - PVT by Google [Yang, Guliani, Beaufays and Motta, 2021]
  - Masked NNs [Mohtashami, Jaggi, Stich, 2021] [More information at <https://akyrillidis.github.io/ist/>]
  - General theory [Zhou, Lan, Venkataramani and Ding, 2022]
  - Federated Dropout [Wen, Jean, and Huang, 2022]
  - Federated Pruning (Google) [Lin et al., 2022]

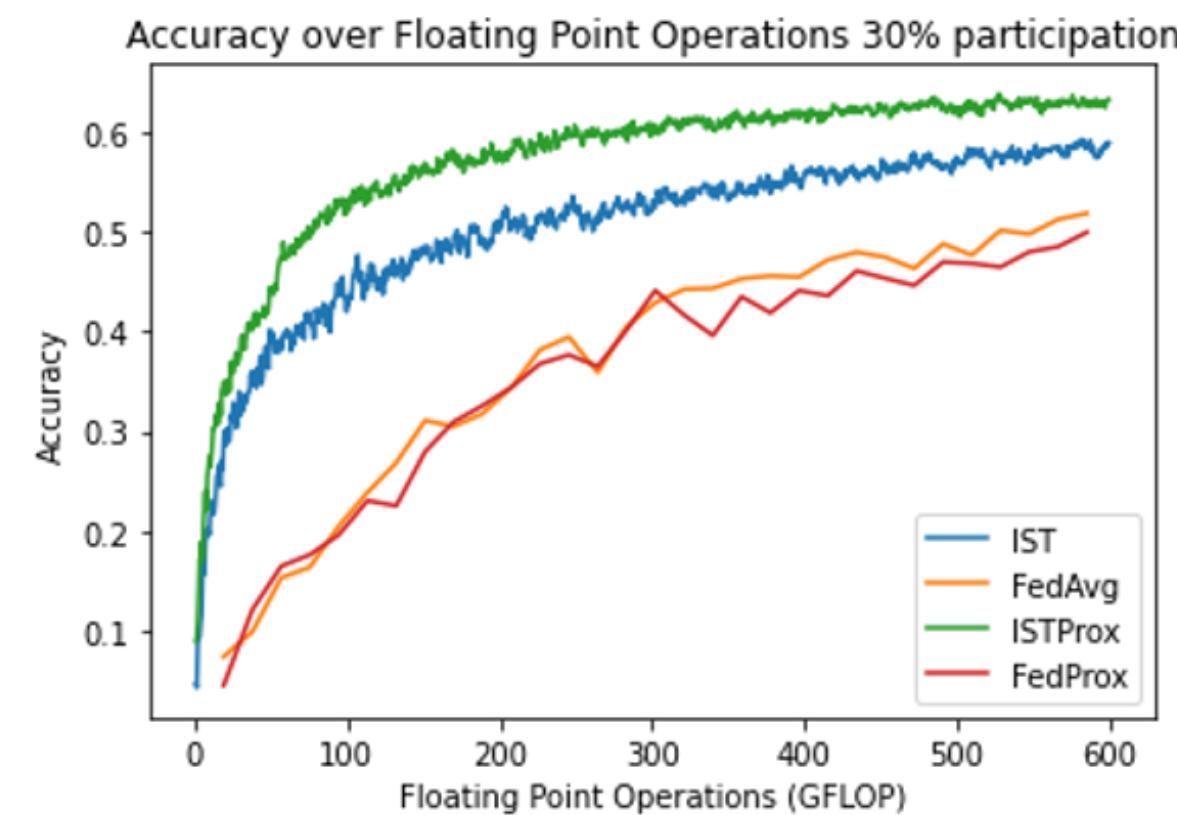
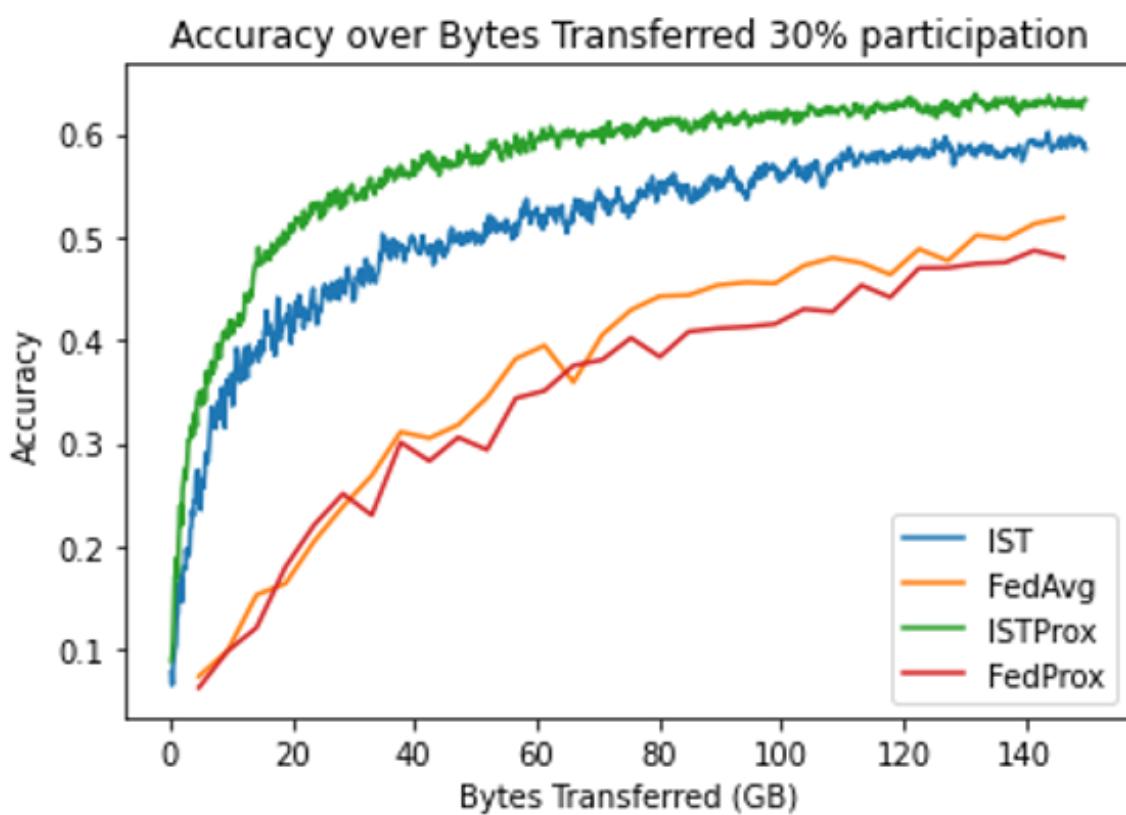
# Take home message



- A.. different distributed protocol
- Potential impact on communication, compute requirements
- We need a clearer view of large-scale models with hundreds of workers
- Unifies well existing models with theory

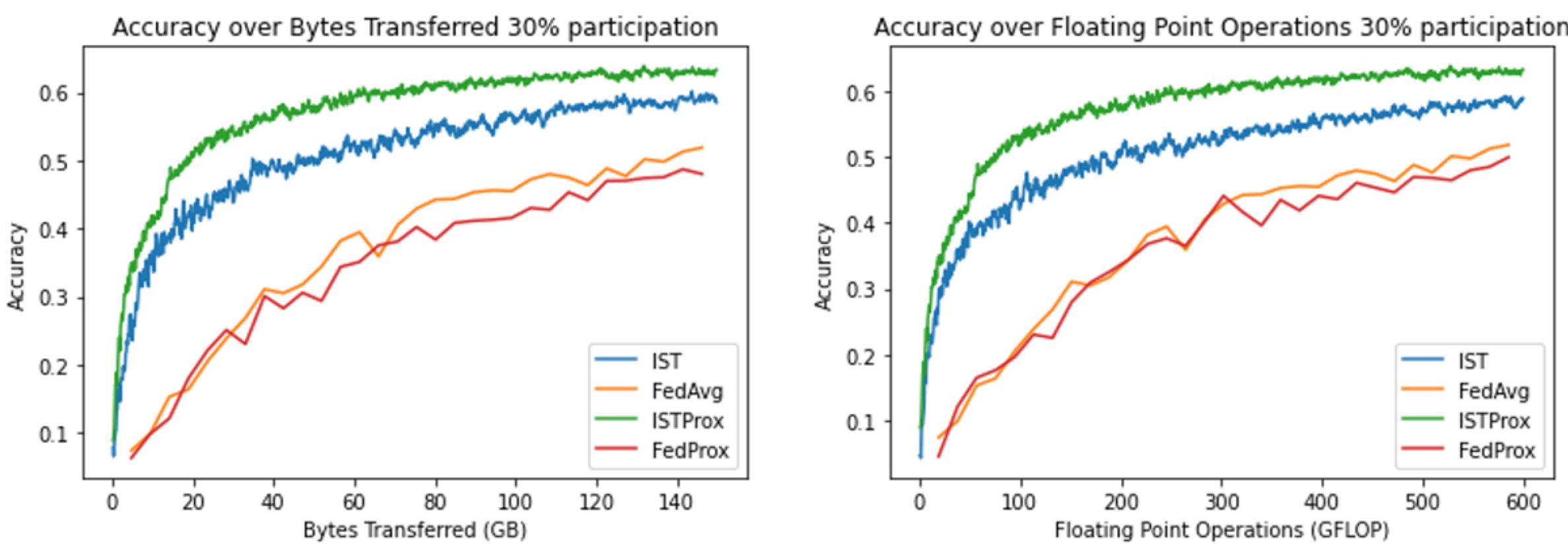
# Where to go from here?

- IST + FL

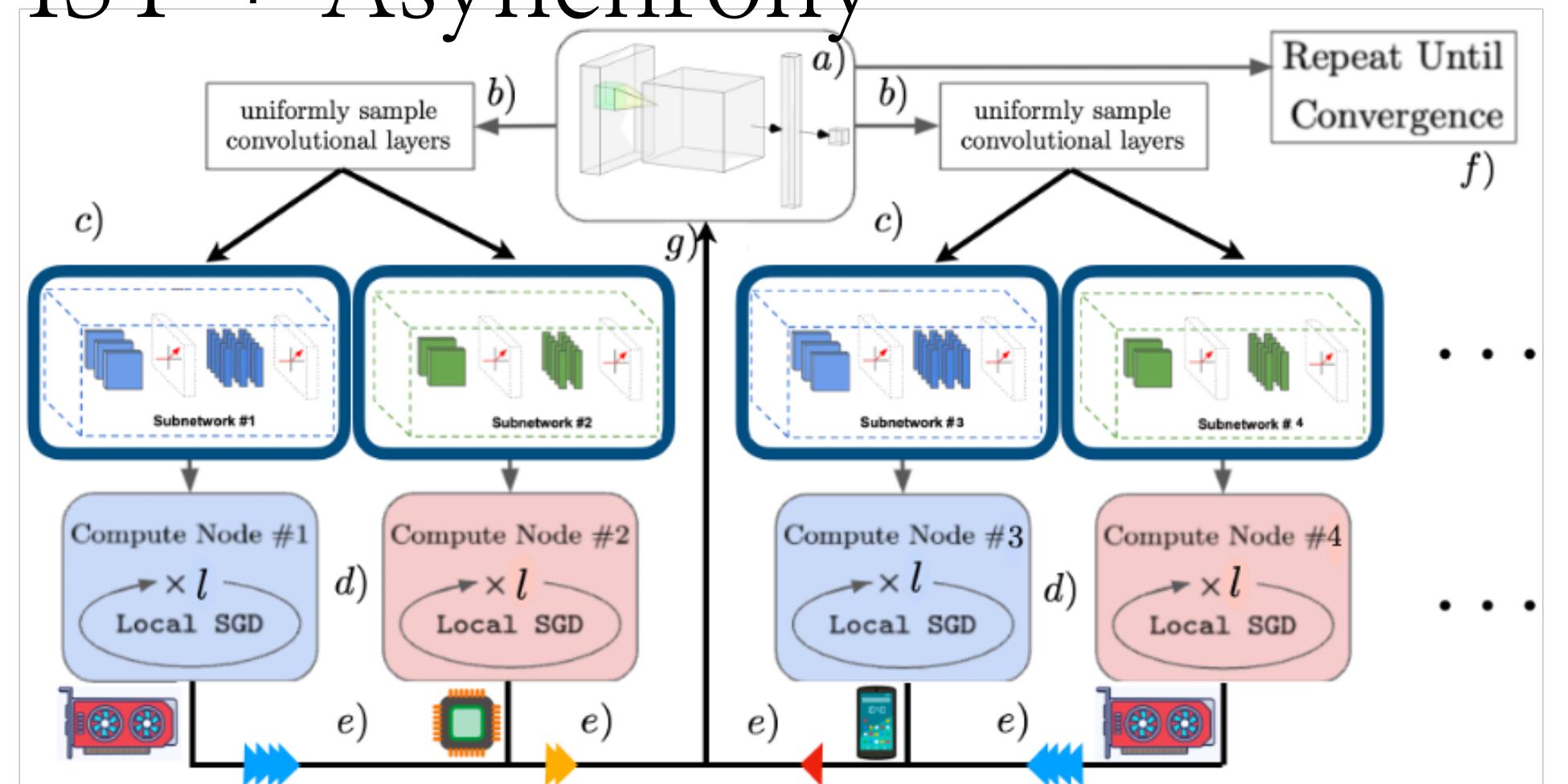


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- IST + FL

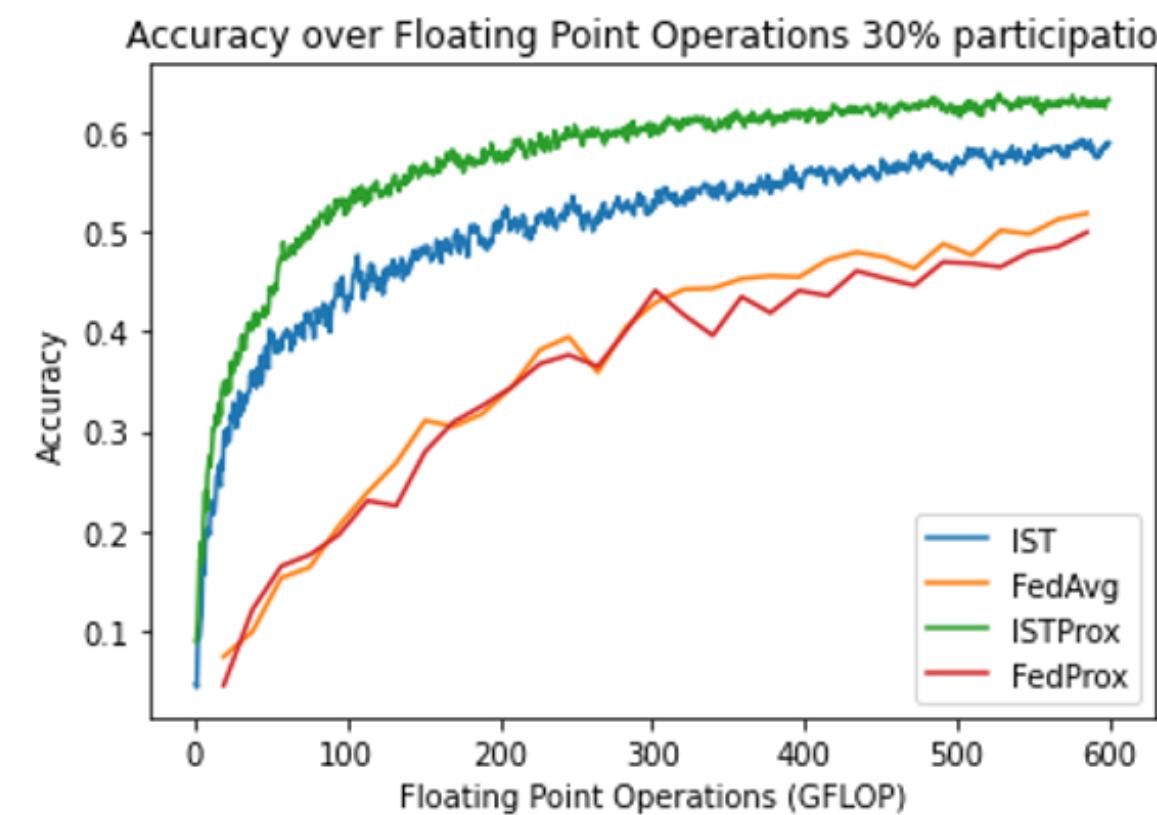
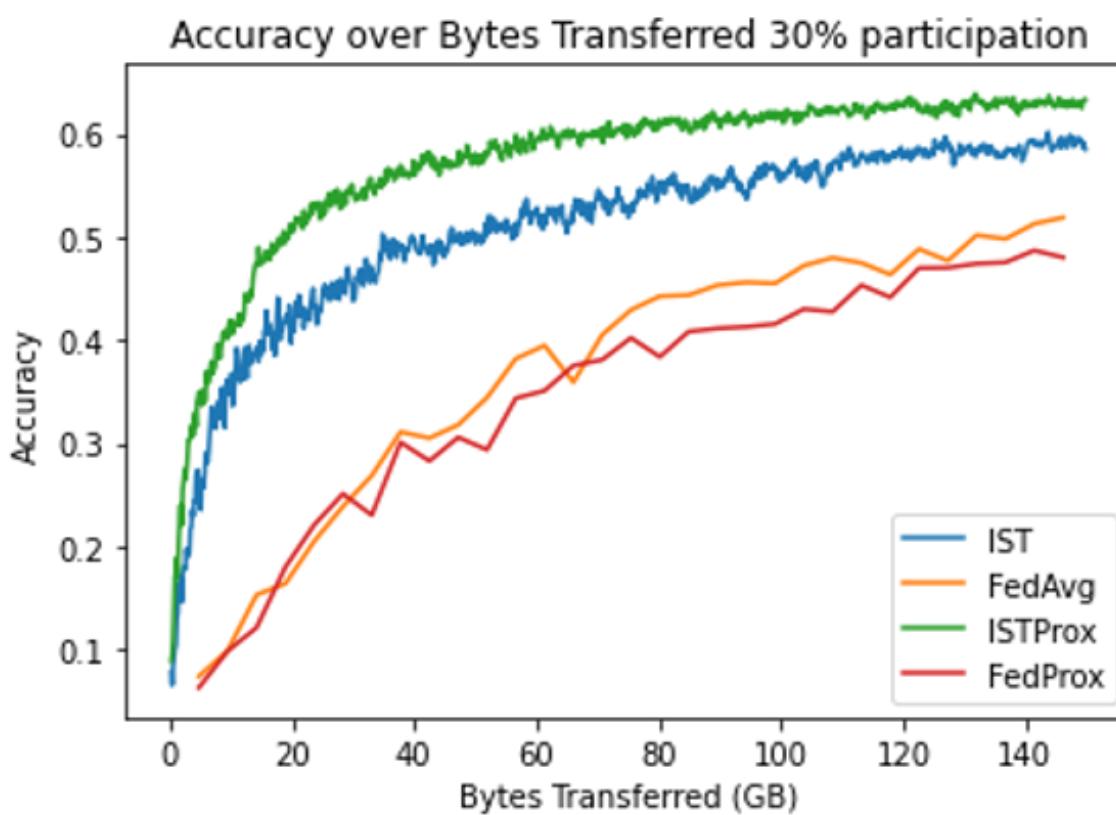


- IST + Asynchrony



# Where to go from here?

- IST + FL

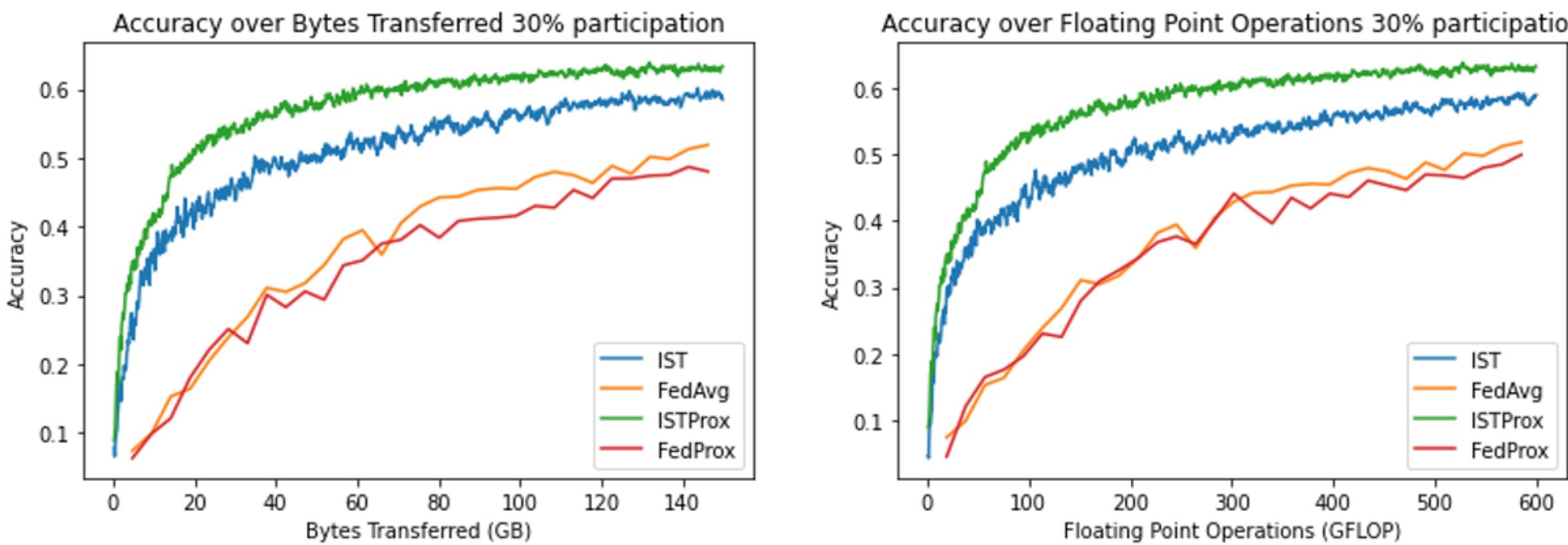


- IST + Asynchrony

$$\begin{aligned} \mathbb{E}_{\mathbf{M}_t} [\|\mathbf{u}_{t+1} - \mathbf{y}\|_2^2] &\leq \left(1 - \frac{\theta\eta\lambda_0}{4}\right)^t \|\mathbf{u}_0 - \mathbf{y}\|_2^2 \\ &+ O\left(\frac{\theta\eta\lambda_0^3\xi^2\kappa^2E^2}{n^2} + \frac{\xi^2(1-\xi)^2\theta\eta n^3\kappa^2d}{m\lambda_0} + \frac{\eta^2\theta^2n\kappa^2\lambda_0\xi^4E^2}{m^4} + \frac{\xi^2(1-\xi)^2\theta^2\eta^2n^2\kappa^2d}{m^3\lambda_0}\right. \\ &\quad \left.+ \frac{\xi^2(1-\xi)^2\theta^2\eta^2\kappa^2\lambda_0E^2}{m^3} + \frac{\xi^2(1-\xi)^2\theta^2\eta^2n^2\kappa^2d}{m^2\lambda_0} + \frac{n\kappa^2(\theta-\xi^2)}{S}\right) \end{aligned}$$

# Where to go from here?

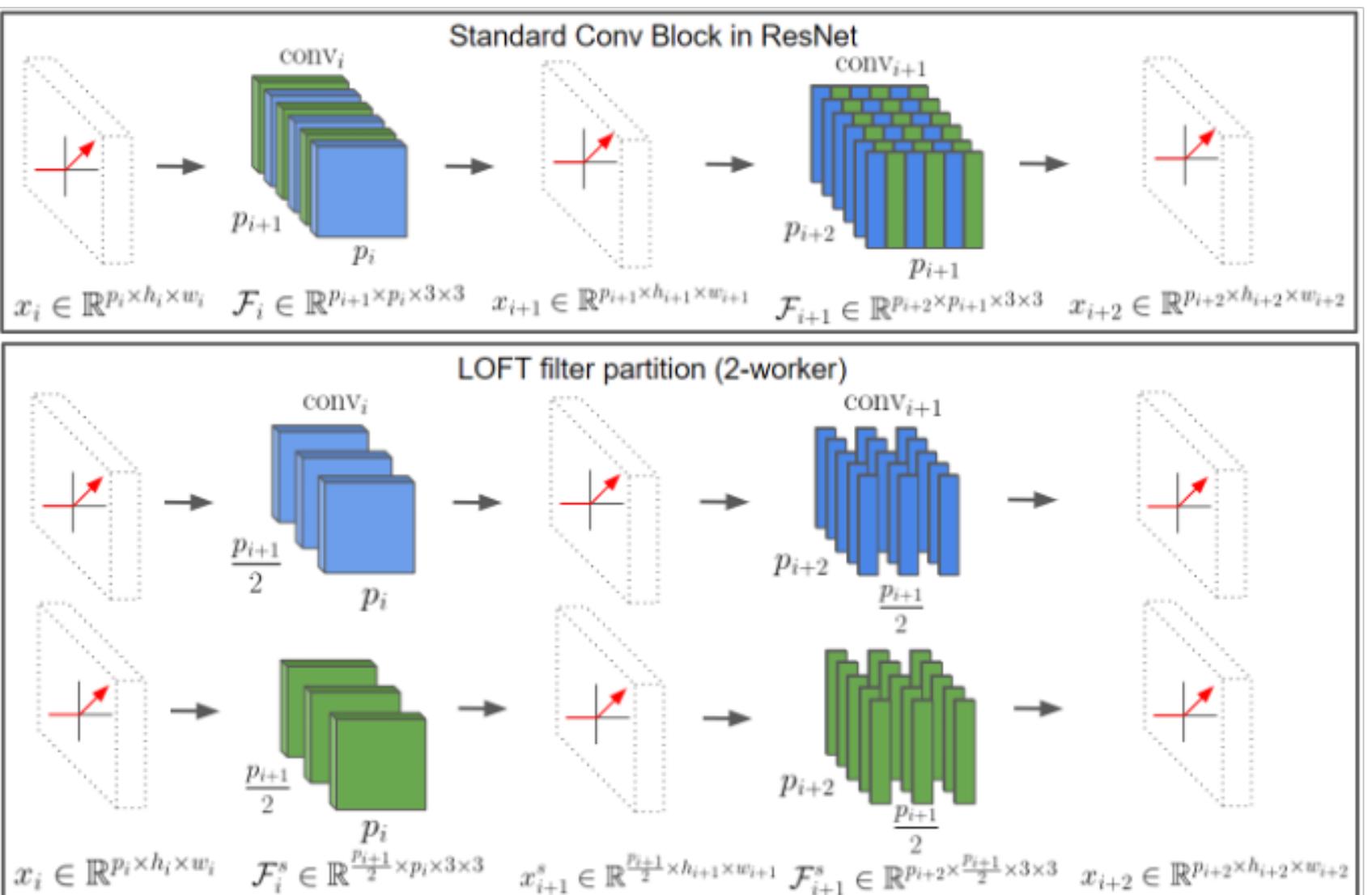
- IST + FL



- IST + Asynchrony

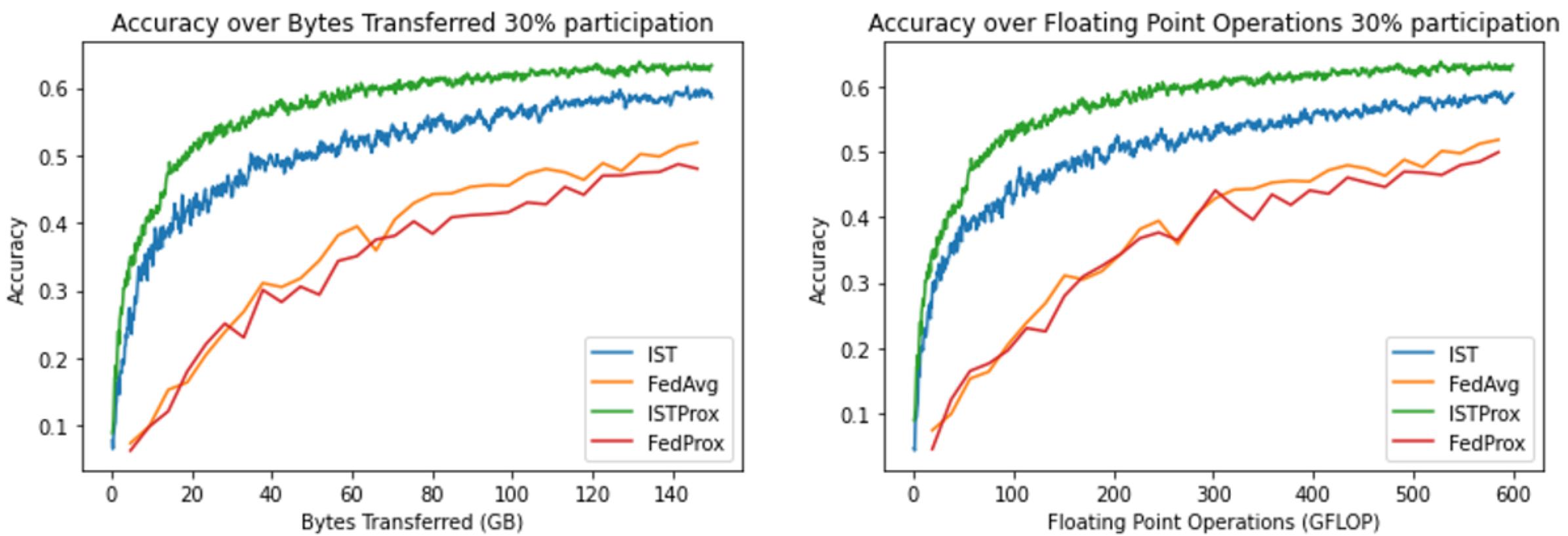
$$\begin{aligned} \mathbb{E}_{\mathbf{M}_t} [\|\mathbf{u}_{t+1} - \mathbf{y}\|_2^2] &\leq \left(1 - \frac{\theta\eta\lambda_0}{4}\right)^t \|\mathbf{u}_0 - \mathbf{y}\|_2^2 \\ &+ O\left(\frac{\theta\eta\lambda_0^3\xi^2\kappa^2E^2}{n^2} + \frac{\xi^2(1-\xi)^2\theta\eta n^3\kappa^2d}{m\lambda_0} + \frac{\eta^2\theta^2n\kappa^2\lambda_0\xi^4E^2}{m^4} + \frac{\xi^2(1-\xi)^2\theta^2\eta^2n^2\kappa^2d}{m^3\lambda_0}\right. \\ &\quad \left.+ \frac{\xi^2(1-\xi)^2\theta^2\eta^2\kappa^2\lambda_0E^2}{m^3} + \frac{\xi^2(1-\xi)^2\theta^2\eta^2n^2\kappa^2d}{m^2\lambda_0} + \frac{n\kappa^2(\theta-\xi^2)}{S}\right) \end{aligned}$$

- IST + LTH



# Where to go from here?

- IST + FL



- IST + Asynchrony

$$\begin{aligned} \mathbb{E}_{\mathbf{M}_t} [\|\mathbf{u}_{t+1} - \mathbf{y}\|_2^2] &\leq \left(1 - \frac{\theta\eta\lambda_0}{4}\right)^t \|\mathbf{u}_0 - \mathbf{y}\|_2^2 \\ &+ O\left(\frac{\theta\eta\lambda_0^3\xi^2\kappa^2E^2}{n^2} + \frac{\xi^2(1-\xi)^2\theta\eta n^3\kappa^2d}{m\lambda_0} + \frac{\eta^2\theta^2n\kappa^2\lambda_0\xi^4E^2}{m^4} + \frac{\xi^2(1-\xi)^2\theta^2\eta^2n^2\kappa^2d}{m^3\lambda_0}\right. \\ &\quad \left.+ \frac{\xi^2(1-\xi)^2\theta^2\eta^2\kappa^2\lambda_0E^2}{m^3} + \frac{\xi^2(1-\xi)^2\theta^2\eta^2n^2\kappa^2d}{m^2\lambda_0} + \frac{n\kappa^2(\theta-\xi^2)}{S}\right) \end{aligned}$$

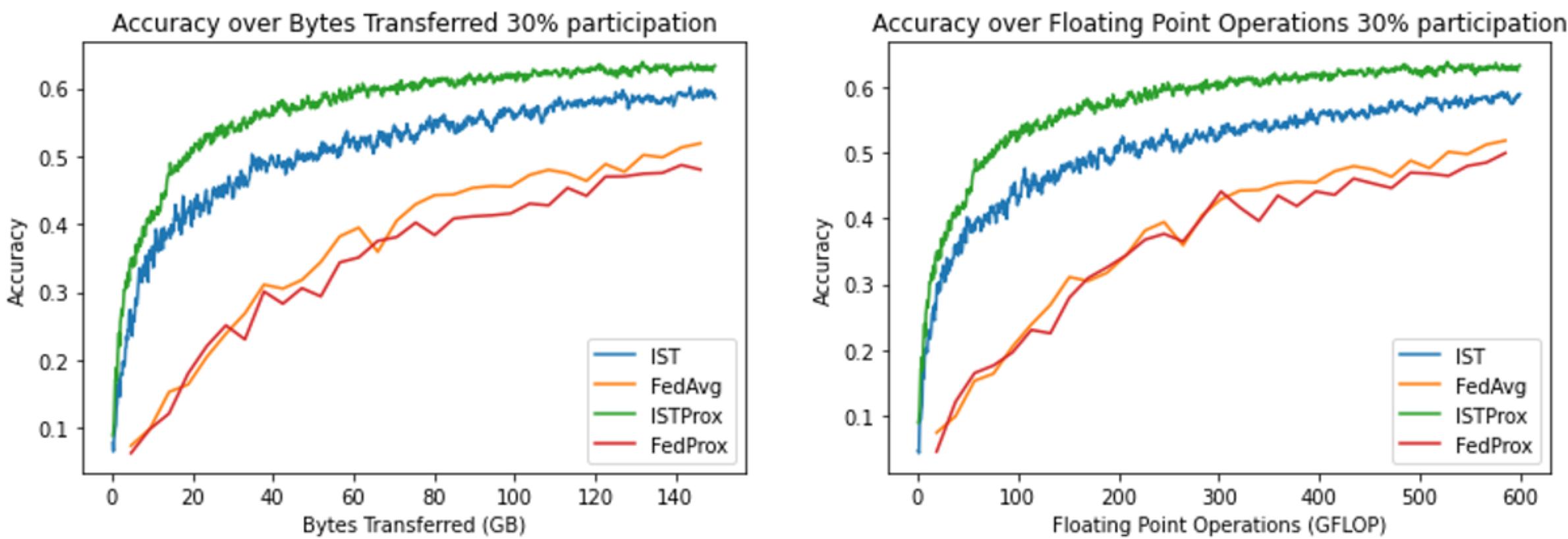
- IST + LTH

**Theorem 3.** Let  $f(\cdot, \cdot)$  be a one-hidden-layer CNN with the second layer weight fixed. Assume the number of hidden neurons satisfies  $m = \Omega\left(\frac{n^4K^2}{\lambda_0^4\delta^2} \max\{n, d\}\right)$  and the step size satisfies  $\eta = O\left(\frac{\lambda_0}{n^2}\right)$ : Let Assumptions 1 and 2 be satisfied. Then, with probability at least  $1 - O(\delta)$  we have:

$$\mathbb{E}_{[\mathbf{M}_T]} \left[ \left\| \mathbf{W}_T - \hat{\mathbf{W}}_T \right\|_F^2 \right] + \eta \sum_{t=0}^{T-1} \mathbb{E}_{[\mathbf{M}_T]} \left[ \left\| f(\mathbf{X}, \mathbf{W}_t) - f(\mathbf{X}, \hat{\mathbf{W}}_t) \right\|_2^2 \right] \leq O\left(\frac{n^2\sqrt{d}}{\lambda_0^2\kappa m^{\frac{1}{4}}\sqrt{\delta}} + \frac{2\eta^2 T \theta^2 (1-\xi)\lambda_0}{S}\right).$$

# Where to go from here?

- IST + FL



- IST + Asynchrony

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- IST + LTH

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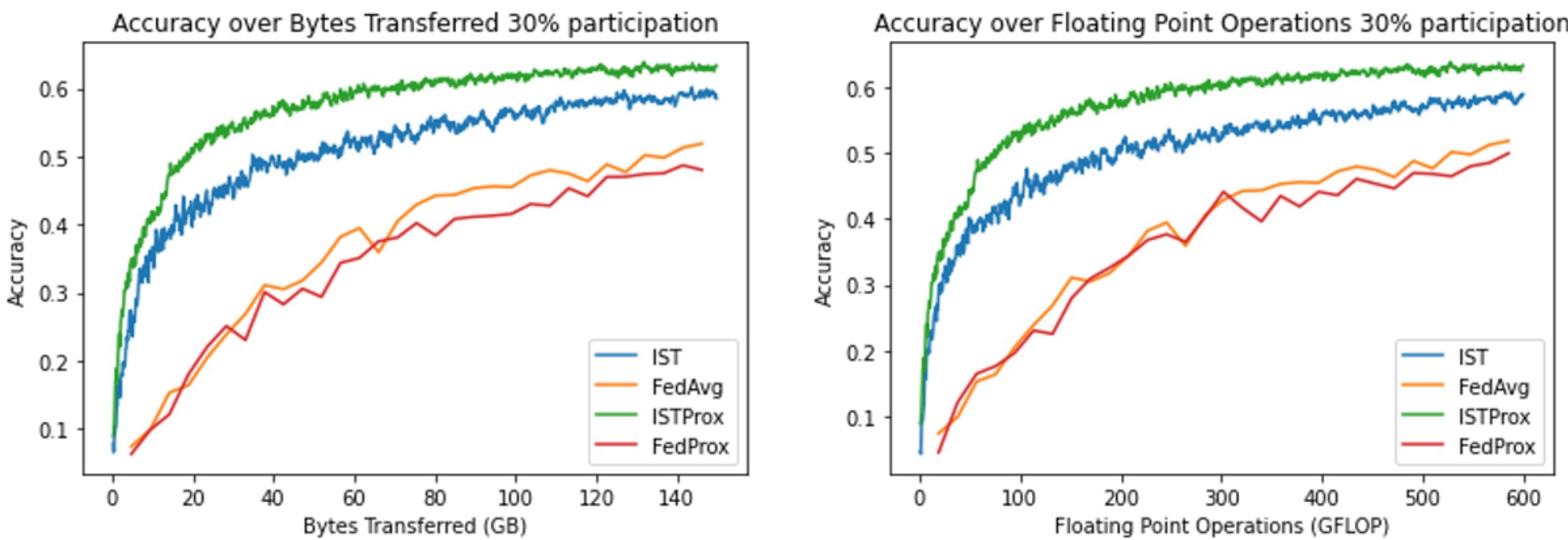
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- IST + modern NNs (Transformers)

(Ongoing)

# Where to go from here?

- IST + FL



- IST + Asynchrony

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- IST + modern NNs (Transformers)

(Ongoing)

Thank you!